

# Sample Spaces, Subsets and Basic Probability

CCM2 Unit 6: Probability

# Sample Space

- **Sample Space:** The **set** of all possible outcomes of an experiment.
- List the sample space,  $S$ , for each of the following:
  - a. Tossing a coin
    - $S = \{H, T\}$
  - b. Rolling a six-sided die
    - $S = \{1, 2, 3, 4, 5, 6\}$
  - c. Drawing a marble from a bag that contains two red, three blue and one white marble
    - $S = \{\text{red}, \text{red}, \text{blue}, \text{blue}, \text{blue}, \text{white}\}$

# Intersections and Unions of Sets

- The **intersection** of two sets ( $A \cap B$ ) is the set of all elements in both set A **AND** set B.
- The **union** of two sets ( $A \cup B$ ) is the set of all elements in set A **OR** set B.
- Example: Given the following sets, find  $A \cap B$  and  $A \cup B$

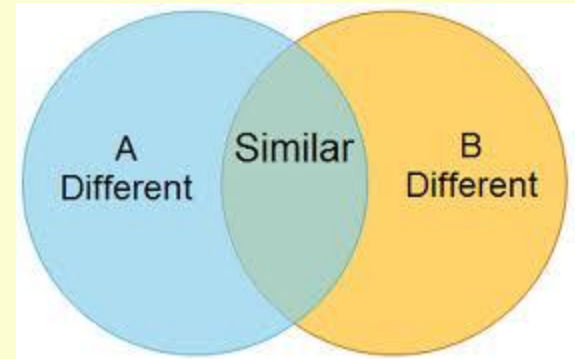
$$A = \{1,3,5,7,9,11,13,15\} \quad B = \{0,3,6,9,12,15\}$$

$$A \cap B = \{3,9,15\}$$

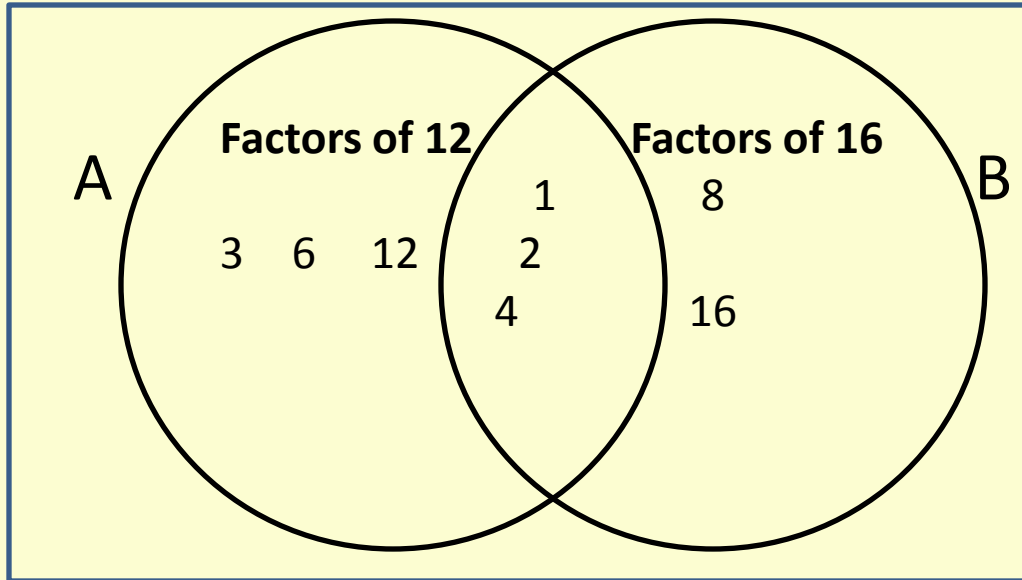
$$A \cup B = \{0,1,3,5,6,7,9,11,12,13,15\}$$

# Venn Diagrams

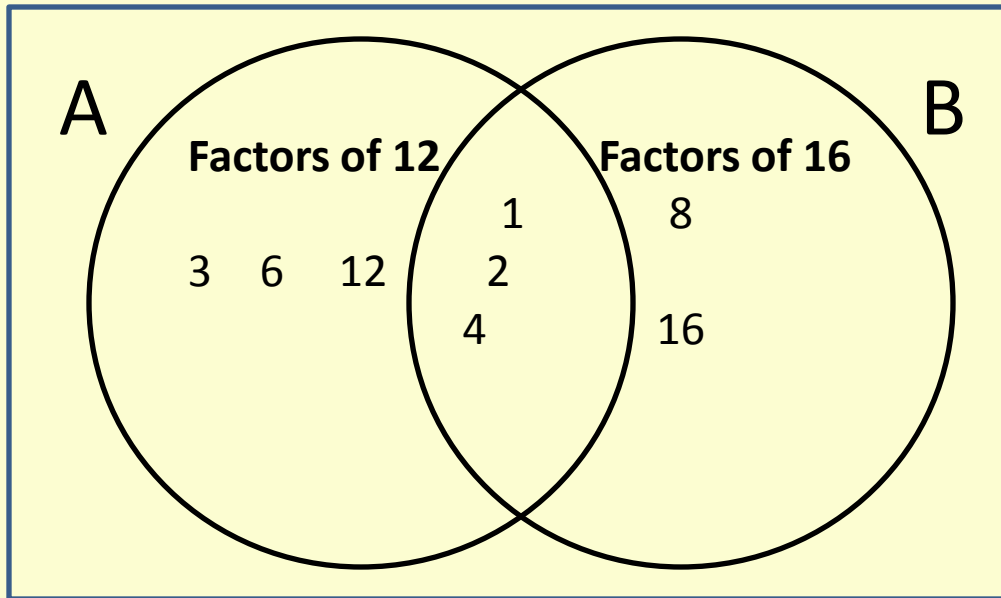
- Sometimes drawing a diagram helps in finding intersections and unions of sets.
- A **Venn Diagram** is a visual representation of sets and their relationships to each other using overlapping circles. Each circle represents a different set.



Use the Venn Diagram to answer the questions below:



1. What are the elements of set A?  
 $\{1,2,3,4,6,12\}$
2. What are the elements of set B?  
 $\{1,2,4,8,16\}$
3. Why are 1, 2, and 4 in both sets?



4. What is  $A \cap B$ ?

$\{1, 2, 4\}$

5. What is  $A \cup B$ ?

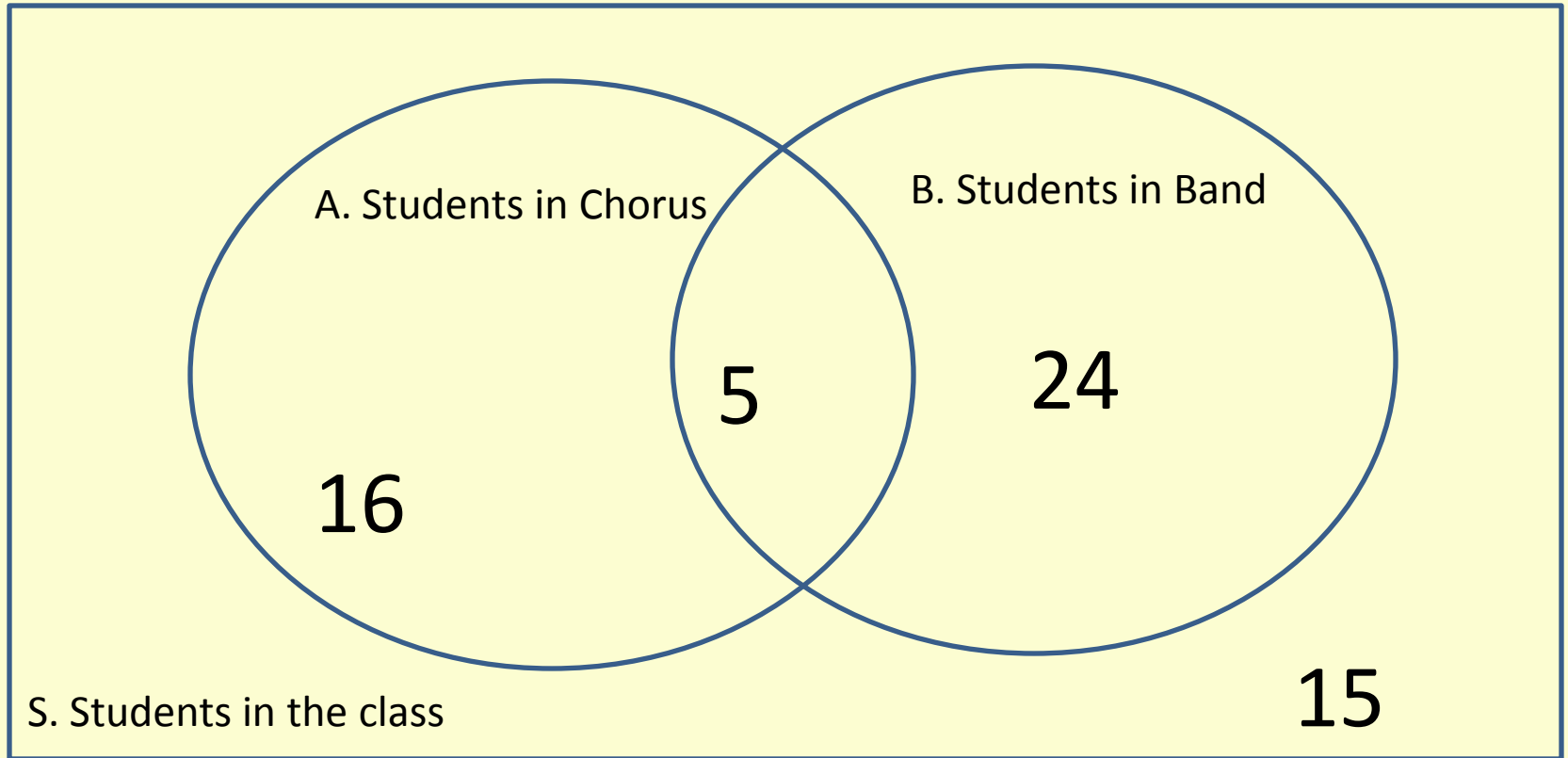
$\{1, 2, 3, 4, 6, 8, 12, 16\}$

In a class of 60 students, 21 sign up for chorus, 29 sign up for band, and 5 take both. 15 students in the class are not enrolled in either band or chorus.

6. Put this information into a Venn Diagram. If the sample space,  $S$ , is the set of all students in the class, let students in chorus be set  $A$  and students in band be set  $B$ .

7. What is  $A \cup B$ ?

8. What is  $A \cap B$ ?



$$A \cup B = \{45\}$$

$$A \cap B = \{5\}$$

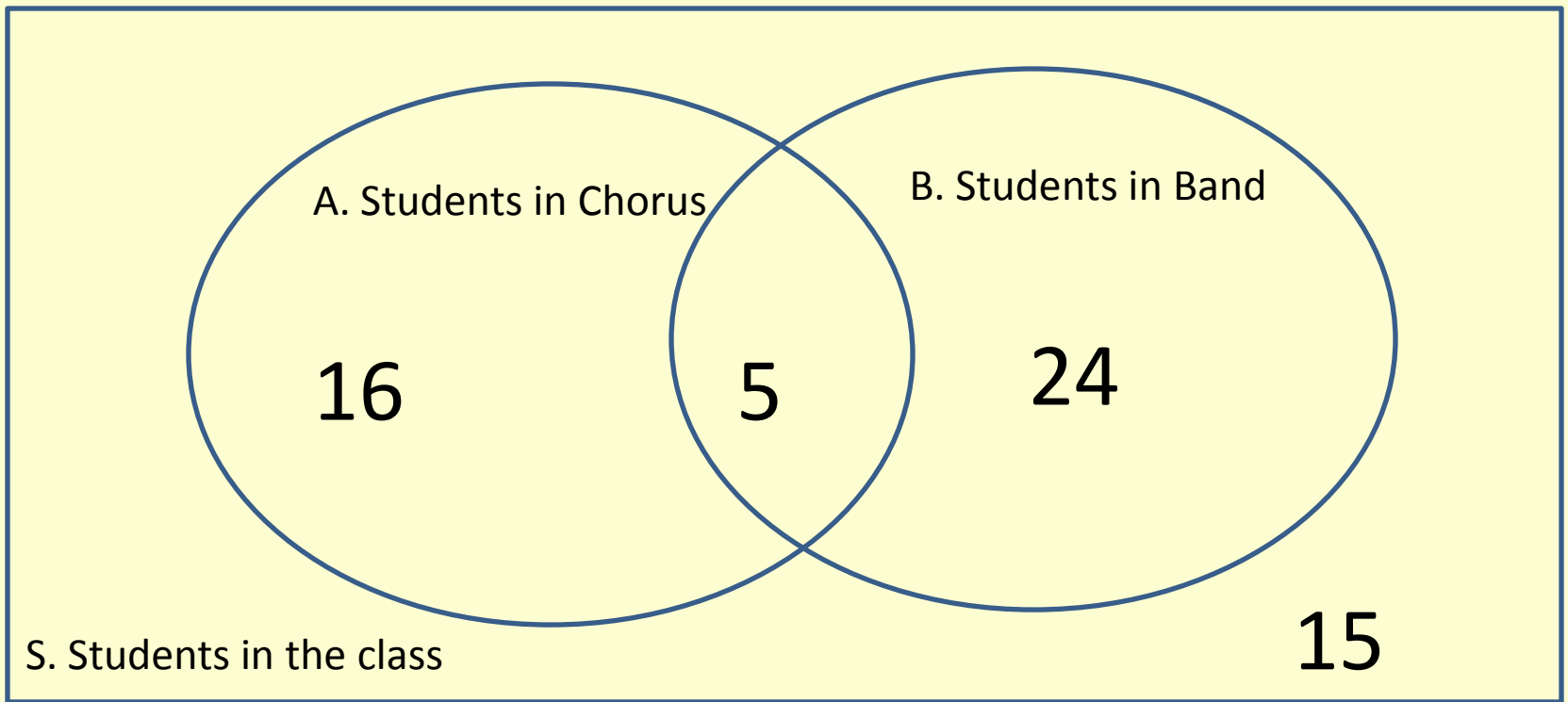


# Compliment of a set

- The **complement** of a set is the set of all elements **NOT** in the set.
  - The compliment of a set, A, is denoted as  $A^C$
- Ex:  $S = \{\dots-3,-2,-1,0,1,2,3,4,\dots\}$   
 $A = \{\dots-2,0,2,4,\dots\}$

If A is a subset of S, what is  $A^C$ ?

$$A^C = \{-3,-1,1,3,5,\dots\}$$



9. What is  $A^C$ ?       $B^C$ ?  
    {39}                      {31}
10. What is  $(A \cap B)^C$ ?  
    {55}
11. What is  $(A \cup B)^C$ ?  
    {15}

# Basic Probability

- Probability of an event occurring is:

$$P(E) = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$$

- We can use sample spaces, intersections, unions, and compliments of sets to help us find probabilities of events.
  - Note that  **$P(A^C)$**  is every outcome **except (or not)**  $A$ , so we can find  $P(A^C)$  by finding  $1 - P(A)$ 
    - Why do you think this works?

An experiment consists of tossing three coins.

12. List the sample space for the outcomes of the experiment.

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

13. Find the following probabilities:

a. P(all heads)

$1/8$

b. P(two tails)

$3/8$

c. P(no heads)

$1/8$

d. P(at least one tail)

$7/8$

e. How could you use compliments to find d?

The compliment of at least one tail is no tails, so you could do  
 $1 - P(\text{no tails}) = 1 - 1/8 = 7/8$

A bag contains six red marbles, four blue marbles, two yellow marbles and 3 white marbles. One marble is drawn at random.

14. List the sample space for this experiment.

$\{r, r, r, r, r, r, b, b, b, b, y, y, w, w, w\}$

15. Find the following probabilities:

a.  $P(\text{red})$

$\frac{2}{5}$

b.  $P(\text{blue or white})$

$\frac{7}{15}$

c.  $P(\text{not yellow})$

$\frac{13}{15}$

(Note that we could either count all the outcomes that are not yellow or we could think of this as being  $1 - P(\text{yellow})$ . Why is this?)

A card is drawn at random from a standard deck of cards. Find each of the following:

16.  $P(\text{heart})$

$$13/52 \text{ or } \frac{1}{4}$$

17.  $P(\text{black card})$

$$26/52 \text{ or } \frac{1}{2}$$

18.  $P(2 \text{ or jack})$

$$8/52 \text{ or } \frac{2}{13}$$

19.  $P(\text{not a heart})$

$$39/52 \text{ or } \frac{3}{4}$$

# Odds

- The **odds** of an event occurring are equal to the ratio of favorable outcomes to **unfavorable outcomes**.

$$\text{Odds} = \frac{\text{Favorable Outcomes}}{\text{Unfavorable Outcomes}}$$

20. The weather forecast for Saturday says there is a 75% chance of rain. What are the odds that it will rain on Saturday?

- What does the 75% in this problem mean?
  - In 100 days where conditions were the same as Saturday, it rained on 75 of those days.
- The favorable outcome in this problem is that it rains:
  - 75 favorable outcomes, 25 unfavorable outcomes
  - Odds(rain) =  $75/25$  or  $3/1$
- Should you make outdoor plans for Saturday?



21. What are the odds of drawing an ace at random from a standard deck of cards?

$$\begin{aligned}\text{Odds(ace)} &= 4/48 \\ &= 1/12\end{aligned}$$