Name Key

Greatest Integer Function

The Greatest Integer Function the "Step Function"

$f(x) = [\![x]\!]$ or $[x]$ or $\lfloor x \rfloor$

no fractions/decimals

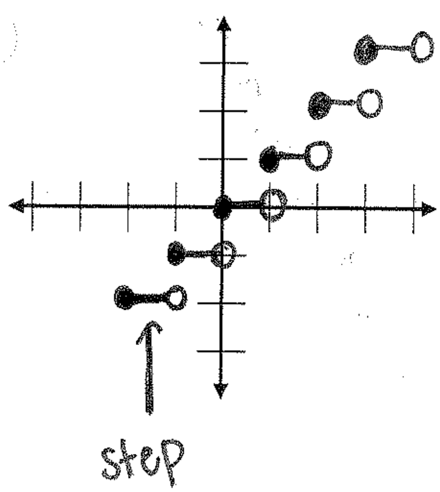
This function takes the input and finds the greatest integer closest to that number without going over.

Examples:	*Answers*	*Examples:*	*Answers:*
1. $[7.35] =$	7	3. $[-2.5]$	−3
2. $\left[\frac{4}{3}\right] =$	1	4. $\left[-\frac{10}{5}\right]$	−2

Graphing the Greatest Integer Function

The greatest integer function got its nickname, the step function, from its graph.

$f(x) = [x]$



step length: 1
step height: 1

x	$f(x)$
-2.00	−2
-1.75	−2
-1.5	−2
-1.25	−2
-1.00	−1
-0.75	−1
-0.5	−1
-0.25	−1
0.00	0
0.25	0
0.5	0
0.75	0
1.00	1
1.25	1
1.5	1

● → part of the solution

○ → not part of the solution

Transformations of the Greatest Integer Function

Don't forget the transformations do not change!

*b → horizontal stretch/compression.

*if b is negative, the graph is reflected over the y-axis

Graphing Form: $y = a[\![b(x-h)]\!]+k$

So (h, k) is a starting point for your steps.

$y = [\![x]\!]$

The length of your steps is $1/b$.

The space between your steps (vertically) is a.

If a is positive the steps go up from left to right.

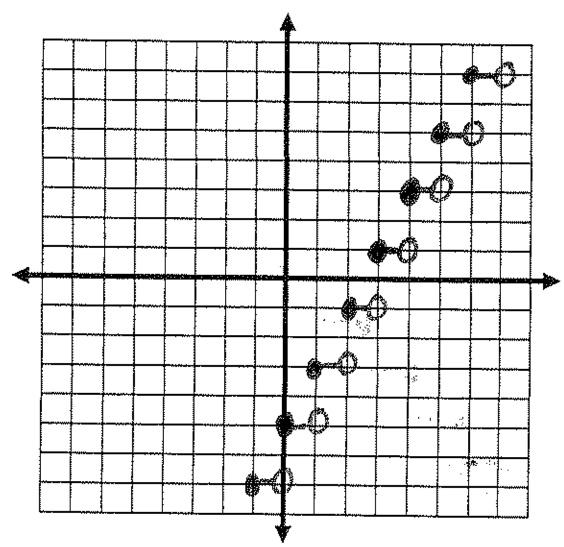
and if a is negative then the steps go down from left to right.

Example: Graph $f(x) = 2[\![x-3]\!]+1$

Start $(3, 1)$

Step length $1/1 = 1$

Step height 2



if there is a GCF, factor it out first!

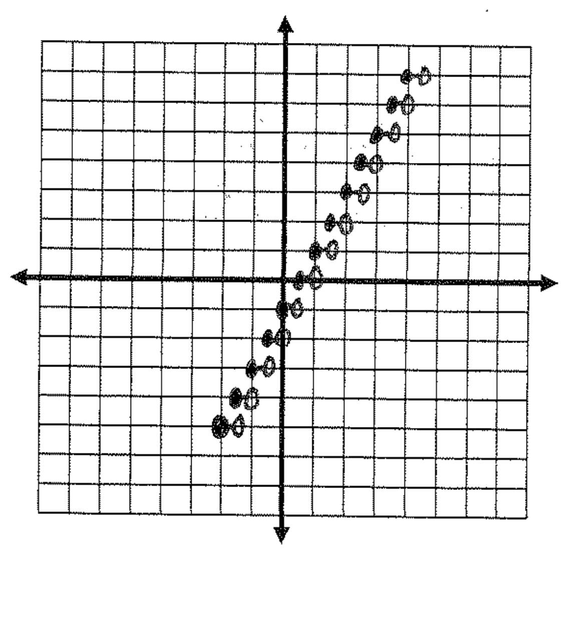
Example 2: Graph $y = [\![2x+4]\!]-5$

*Get in graphing form! $y = [\![2(x+2)]\!]-5$

Start $(-2, -5)$

Step length $1/2$

Step height 1



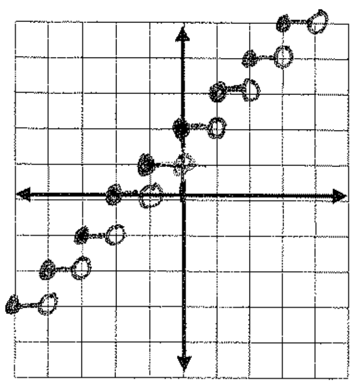
Greatest Integer Function Worksheet Name Key

Evaluate the following:

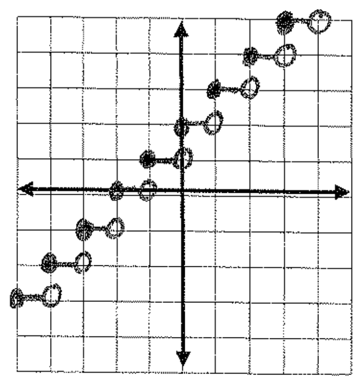
1. $\lfloor 7.1 \rfloor$ 7
2. $\lfloor 1.8 \rfloor$ 1
3. $\lfloor \pi \rfloor$ 3
4. $\lfloor -6.8 \rfloor$ -7
5. $\lfloor -2.1 \rfloor$ -3
6. $\lfloor 0 \rfloor$ 0

Graph the translation in 7 and 8. Then describe the translations and how they differ.

7. $f(x) = \lfloor x \rfloor + 2$



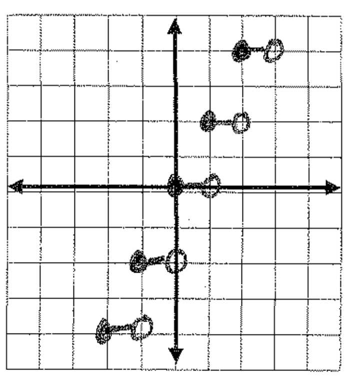
8. $g(x) = \lfloor x + 2 \rfloor$



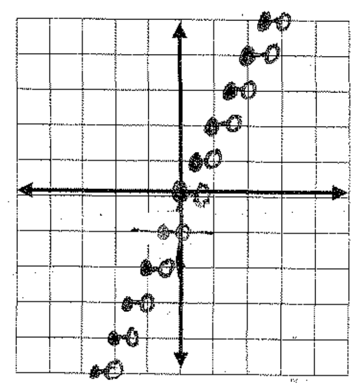
Because they are steps, translating left two & up two, actually produce the same graph.

Graph the dilations in 9 and 10. Then describe the dilations and they differ.

9. $f(x) = 2\lfloor x \rfloor$



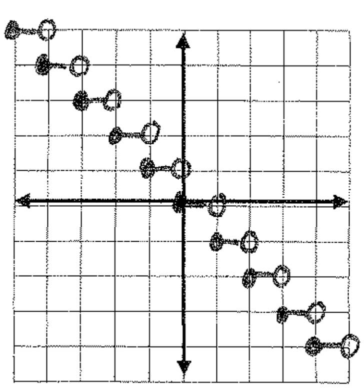
10. $g(x) = \lfloor 2x \rfloor$



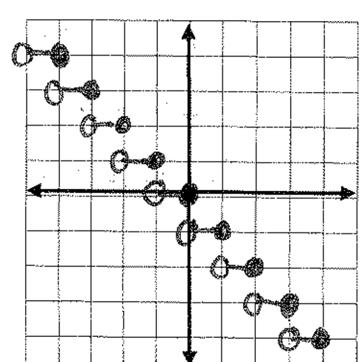
#9 is a vertical stretch (steps are farther apart) while #10 has a horizontal effect & makes the steps shorter.

Graph the reflections in 11 and 12. Then describe the reflection and they differ

11. $f(x) = -[[x]]$



12. $g(x) = [[-x]]$



#11 reflects the graph over the x-axis, while #12 reflects it over the y-axis. For both, the steps now go down from left to right instead of up

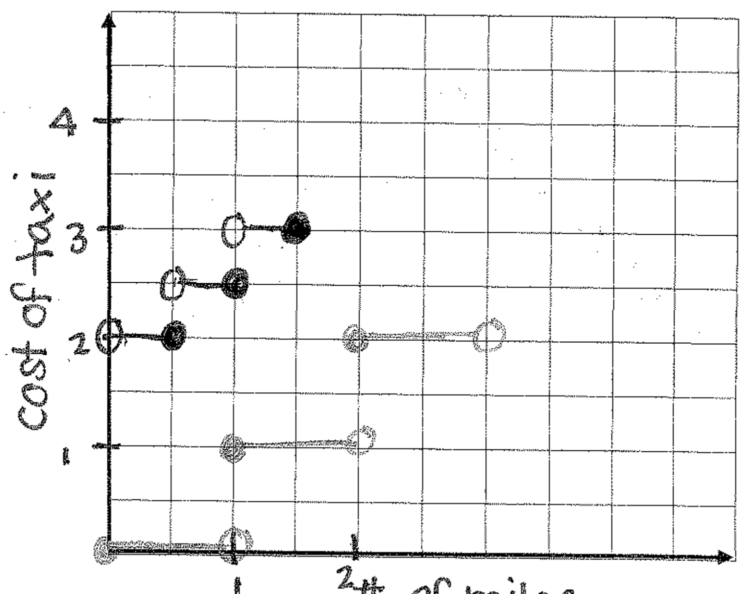
Real World Application of Step Functions:

Prior to September, 2000, taxi fares from Washington DC to Maryland were as follows:
\$2.00 for up to and including the first ½ mile, + \$0.50 for each additional ½ mile increment.

Describe the independent and dependent variables and explain your choices.

Dependent → cost of taxi ride Independent → # of miles

Graph the fares for the first 2 miles: *(Make sure to label the axes.)*



Explain why this is a step function.

How is it different from the greatest integer parent function $f(x) = \lfloor x \rfloor$

The open circle is on the left. Its moved up 2. Length of the step is ½ & height of the step is ½.

★ Write the function that is modeled by this graph. ★

$y = -\frac{1}{2}[\![-2(x-.5)]\!]+2$

Preview for tomorrow: Write the piecewise function for 0 to 2 miles.

$$f(x) = \begin{cases} \$2.00 \quad \textit{if} & 0 < x \le 1/2 \\ \$2.50 & 1/2 < x \le 1 \\ \$3.00 & 1 < x \le 1.5 \end{cases}$$