GUIDED NOTES: Solving Radical Equations

KEY

Team Name (optional):

Your Name:

Partner(s):

Task 1: In this activity, we will solve the equation $\sqrt{3x-2}+2=x$ and try to understand the results.

Step 1. Subtract 2 from each side of the equation. Show your answer in the space below. Your answer should be an equation. We have placed this equation in row 4, column 1 of the table given on the next page. Check to see if you have the same result.

$$\sqrt{3x-2}+2=x$$
$$-2 \quad -2$$
$$\sqrt{3x-2}=x-2$$

Step 2: Square both sides of the equation you got in step one. Recall that you need to distribute when squaring a binomial such as $(a+b)^2$. Show your work in the space below. We have placed the answer in row 3, column 1 of the table provided. Check to see if you have the same result.

$$(\sqrt{3x-2})^2=(x-2)^2$$
$$3x-2=(x-2)(x-2)$$
$$3x-2=x^2-2x-2x+4$$
$$3x-2=x^2-4x+4$$

Step 3: Set the quadratic equation equal to zero (i.e., move everything to one side of the equation). We have placed the answer in row 2, column 1, of the table provided. Check to see if you got the same answer.

$$3x-2=x^2-4x+4$$
$$-3x+2 \quad -3x+2$$
$$0=x^2-7x+6$$

Step 4: Solve the resulting quadratic equation for x. (Factoring should be the easiest method.) We have placed the solutions in row 1, columns 2 and 3, of the table provided. Check to see if your answers agree.

By Factoring

$-6 \cdot -1 = 6$, $-6 + -1 = -7$

	x	-1
x	x^2	$-1x$
-6	$-6x$	$+6$

$(x-6)(x-1)=0$

$x=6 \quad x=1$

By Quadratic Formula

$$\frac{+7\pm\sqrt{(-7)^2-4(1)(6)}}{2(1)}$$
$$\frac{7\pm\sqrt{25}}{2}$$
$$\frac{7+5}{2} \text{ or } \frac{7-5}{2}=\frac{12}{2} \text{ or } \frac{2}{2}$$

6 or 1 (1)

Step 5: Complete the remaining parts of the table by checking each solution in each equation. Row 5 column 2 has been done for you, from which we see that $x = 6$ is a solution of the original equation. Are both 6 and 1 solutions to the final equation, $x^2 - 7x + 6 = 0$? Are both solutions to the original equation, $\sqrt{3x-2} + 2 = x$?

	Column 1	Column 2	Column 3
Row 1	Equation	Solution 1: $x = 6$	Solution 2: $x = 1$
Row 2	$0 = x^2 - 7x + 6$	$0 = 6^2 - 7(6) + 6$ $0 = 36 - 42 + 6$ $0 = -6 + 6$ $0 = 0$ TRUE	$0 = 1^2 - 7(1) + 6$ $0 = 1 - 7 + 6$ $0 = -6 + 6$ $0 = 0$ TRUE
Row 3	$3x - 2 = x^2 - 4x + 4$	$3(6) - 2 = (6)^2 - 4(6) + 4$ $18 - 2 = 36 - 24 + 4$ $16 = 12 + 4$ $16 = 16$ TRUE	$3(1) - 2 = 1^2 - 4(1) + 4$ $3 - 2 = 1 - 4 + 4$ $1 = -3 + 4$ $1 = 1$ TRUE
Row 4	$\sqrt{3x-2} = x - 2$	$\sqrt{3(6)-2} = 6 - 2$ $\sqrt{18-2} = 4$ $\sqrt{16} = 4$ $4 = 4$ TRUE	$\sqrt{3(1)-2} = 1 - 2$ $\sqrt{3-2} = -1$ $\sqrt{1} = -1$ $1 = -1$ FALSE
Row 5	$\sqrt{3x-2} + 2 = x$	$\sqrt{3(6)-2} + 2 = 6$ $\sqrt{18-2} + 2 = 6$ $\sqrt{16} + 2 = 6$ $4 + 2 = 6$ TRUE	$\sqrt{3(1)-2} + 2 = 1$ $\sqrt{3-2} + 2 = 1$ $\sqrt{1} + 2 = 1$ $1 + 2 = 8$ FALSE

Step 6: A number that is a solution to the final equation but not the original equation is called an *extraneous solution*. How does an extraneous solution come about? Extraneous solutions can occur when solving square root equations but not when solving linear, exponential or quadratic equations. Hint: To see what's happening, look at the table and determine at which step of solving the radical equation the extraneous solution was introduced. State your observations.

To summarize: When solving an equation involving a square root,

(a) Isolate the square root.
(b) Square each side of the equation to eliminate the square root.
(c) Solve the resulting (linear or quadratic) equation.
(d) Check the *possible* solutions for any extraneous solutions.

Task 2: Solve the following equations. Be sure to eliminate extraneous solutions.

a) $\sqrt{5-x}+10=0$ (Note: Isolate the radical term first.)

$\sqrt{5-x} = -10$

$(\sqrt{5-x})^2 = (-10)^2$

$5-x = +100$

$-5 \quad -5$

$-x = 95$

$x = -95$ ← EXTRANEOUS SOLUTION!

$\sqrt{5--95}+10=0$

$\sqrt{100}+10=0$

$10+10=0$

FALSE

ANSWER IS NO SOLUTION

b) $\sqrt{3x+1}+1=x$

$\sqrt{3x+1} = x-1$

$(\sqrt{3x+1})^2 = (x-1)^2$ I got this by foiling $(x-1)(x-1)$

$3x+1 = x^2-2x+1$

$-3x-1 \quad -3x-1$

$0 = x^2-5x$

Factor out GCF!

$0 = x(x-5)$

$x=0$ & $x-5=0$

$x=5$

CHECK BOTH SOLUTIONS

$\sqrt{3(0)+1}+1=0$

$\sqrt{1}+1=0$

$1+1=0$ FALSE

$x=0$ IS EXTRANEOUS

$\sqrt{3(5)+1}+1=5$

$\sqrt{15+1}+1=5$

$\sqrt{16}+1=5$

$4+1=5$ TRUE!

$x=5$

c) $x+2\sqrt{x-1}=9$

Unit 4B Day 4 HW

Radical Equations Worksheet Answers

Solve the following. Be sure to check for extraneous solutions.

1. $\sqrt{x+2}=3$ CHECK

2. $\sqrt{4x+9}=5$ CHECK

3. $\sqrt[3]{5-11x}=3$ CHECK

4. $\sqrt[3]{x^2-1}=2$ CHECK

5. $\sqrt{x+7}=x-5$ CHECK

6. $\sqrt{x+5}=x-1$ CHECK

7. $\sqrt{2x+15}=x+6$ CHECK

8. $\sqrt{3x+19}=x-3$ CHECK

9. $\sqrt{5x+6}=3+\sqrt{x+3}$ CHECK

10. $\sqrt{x-3}+\sqrt{x+5}=4$ CHECK