

Integrated Advanced Algebra  
 Notes: Solving Rational Equations  
 Textbook: Lesson 3.11, Pages 176 - 177  
 Homework: Worksheet on Solving Rational Equations

**Essential Question: How do you solve rational equations?**

A rational equation is an equation that contains one or more rational expressions. It can have a variable in the numerator and/or the denominator. Our goal when solving a rational equation is to eliminate the fractions and solve the equation for the variable!

Recall that when you graph a rational function, there is a vertical asymptote. This is an x-value that the graph *approaches* but NEVER touches. When you solve rational equations, there are some values for x that must be excluded from the domain because they will make the denominator equal to zero, and dividing by zero is undefined. Any number that causes the denominator to equal zero is called an extraneous solution.

To find the excluded values, set the denominator equal to zero and solve for the variable; the solutions are the excluded values. When solving rational equations, if all solutions of the rational equation are excluded values then there is no solution to the rational equation!

To solve simple rational equations, the cross product property can be utilized to eliminate the fraction leaving a linear equation to solve. **REMEMBER:** Check your final answers to make sure they are not an excluded value!

Examples: Using the cross product property, solve the following equations. Do not forget to determine the excluded values.

1.  $\frac{6}{x} = \frac{3}{7}$  EV: X=0 ↙ Excluded value

$6(7) = 3x$   
 $42 = 3x$   
 $x = 14$

2.  $\frac{4}{x-7} = \frac{6}{x}$  EV: X=7 ; X=0

$4x = 6(x-7)$   $x-7=0$   
 $4x = 6x - 42$   $+7 \quad +7$   
 $-2x = -42$   $x=7$   
 $x = 21$   $x=0$

3.  $\frac{-5}{x+4} = \frac{1}{x+4}$  EV: X=-4

$-5(x+4) = 1(x+4)$   
 $-5x - 20 = x + 4$   
 $-5x = x + 24$   
 $-x \quad -x$   
 $-6x = 24$   
 $x = -4$

**NO SOLUTION**

4.  $\frac{6}{x+5} = \frac{x}{6}$  EV: X=-5

$x(x+5) = 36$   
 $x^2 + 5x = 36$   
 $x^2 + 5x - 36 = 0$   
 $(x-4)(x+9) = 0$   
 $x = 4 \quad x = -9$

~~$9 \times -4$~~   
 $5$   
 $x-4$   

$x^2$	$-4x$
$+9x$	$-36$

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

Sometimes, you will have more than one rational expression on one or both sides of the equation. When this situation occurs the cross product property will not apply. You must find the LCD (least common denominator). By multiplying the entire equation by the LCD, the fractions are eliminated and you are left with an equation to simplify and solve.

Examples: Multiply through by the LCD to solve the following equations. Do not forget to determine the excluded values.

5.  $\frac{2}{x} - 3 = \frac{8}{x}$  EV:  $x=0$

$x \cdot \frac{2}{x} = 2$

LCD:  $x$

$x \left( \frac{2}{x} - 3 \right) = \frac{8}{x} \cdot x$

$2 - 3x = 8$   
 $-2 \quad -2$   
 $-3x = 6$   $x = -2$

6.  $\frac{7x}{x-3} + 4 = \frac{x+1}{x-3}$  EV:  $x=3$

LCD:  $x-3$   
 $(x-3) \left( \frac{7x}{x-3} + 4 \right) = \frac{x+1}{x-3} \cdot (x-3)$

$7x + 4x - 12 = x + 1$   $\circ (0x = 13)$   
 $11x - 12 = x + 1$   
 $-x + 12 = -x + 12$   $x = \frac{13}{10}$

Examples: Solve the rational equation. Do not forget to determine the excluded values.

7.  $\frac{8}{x+8} = \frac{x}{x+2}$  EV:  $x=-8 \div x=-2$

$8(x+2) = x(x+8)$   
 $8x + 16 = x^2 + 8x$   
 $-8x \quad -8x$   
 $\sqrt{16} = \sqrt{x^2}$   
 $x = 4 \text{ or } -4$

8.  $\frac{4}{x+2} + 3 = \frac{9}{x+2}$  EV:  $x=-2$

$\frac{4}{x+2} + 3(x+2) = \frac{9}{x+2}$   
 $4 + 3(x+2) = 9$   $x = -\frac{1}{3}$   
 $4 + 3x + 6 = 9$   
 $3x + 10 = 9$   
 $3x = -1$

9.  $\frac{3x}{x-1} - 2 = \frac{10}{x-1}$  EV:  $x=1$

$\frac{3x}{x-1} - 2(x-1) = \frac{10}{x-1}$   
 $3x - 2(x-1) = 10$   
 $3x - 2x + 2 = 10$   
 $x + 2 = 10$   
 $x = 8$

10.  $\frac{12}{x+2} = \frac{7}{x-3}$  EV:  $x=-2 \div x=3$

$12(x-3) = 7(x+2)$   
 $12x - 36 = 7x + 14$   
 $-7x \quad -7x$   
 $5x - 36 = 14$   
 $+36 \quad +36$   
 $5x = 50$   
 $x = 10$

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Solve the rational equation. Do not forget to determine the excluded values.

**Cross multiply**

1.  $\frac{3}{x} = \frac{2}{x+4}$  EV:  $x=0$ ;  $x=-4$

$3(x+4) = 2x$

$3x + 12 = 2x$

$12 = -x$

$x = -12$

LCD:  $x+2$

3.  $\frac{3}{x+2} + 5 = \frac{4}{x+2}$

EV:  $x = -2$   
 $x+2=0$   
 $x=-2$

$\frac{3}{x+2}(x+2) + 5(x+2) = \frac{4}{x+2}(x+2)$

$3 + 5(x+2) = 4$

$3 + 5x + 10 = 4$

LCD  $5x + 13 = 4$

$5x = -9$

$x = -9/5$

5.  $\frac{2x}{x+4} - 3 = \frac{-12}{x+4}$

EV:  $x = -4$

$\frac{2x}{x+4}(x+4) - 3(x+4) = \frac{-12}{x+4}(x+4)$

$2x - 3(x+4) = -12$   $-x - 12 = -12$

$2x - 3x - 12 = -12$

$-x = 0$

$x = 0$

7.  $\frac{2}{x-4} + 2 = \frac{6}{x-4}$

EV:  $x = 4$

$\frac{2}{x-4}(x-4) + 2(x-4) = \frac{6}{x-4}(x-4)$

$2 + 2(x-4) = 6$

$2 + 2x - 8 = 6$

$2x - 6 = 6$

$+6 +6$   
 $2x = 12$

$x = 6$

**Cross**

2.  $\frac{x+1}{2x+5} = \frac{2}{x}$  EV:  $x=2.5$ ;  $x=0$

$x(x+1) = 2(2x+5)$

$x^2 + x = 4x + 10$

$-4x - 10$   
 $x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5$   
 $x = -2$

**Cross**

4.  $\frac{6}{x-3} = \frac{x}{18}$  EV:  $x = 3$

$x(x-3) = 6(18)$

$x^2 - 3x = 108$

$x^2 - 3x - 108 = 0$   
 $(x+9)(x-12) = 0$

$x = -9$   
or  
 $x = 12$

**Cross**

6.  $\frac{14}{2-x} = \frac{2}{x}$  EV:  $x = 2$  or  $x = 0$

$14x = 2(2-x)$

$14x = 4 - 2x$

$\frac{16x}{16} = \frac{4}{16}$

$x = \frac{1}{4}$

LCD

8.  $\frac{x+2}{x+1} - x = \frac{-6}{x+1}$  EV:  $x = -1$

$\frac{x+2}{x+1}(x+1) - x(x+1) = \frac{-6}{x+1}(x+1)$

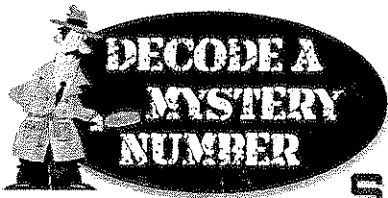
$x+2 - x(x+1) = -6$

$x+2 - x^2 - x = -6$

$-x^2 - x + 8 = -6$   
 $-x^2 - x + 14 = 0$

$x^2 = 8$

$x = 2.8$   
or  $-2.8$



Name/Group Name: \_\_\_\_\_

### SOLVING RATIONAL EQUATIONS

Congratulations, Agent(s)! You have been selected to help investigate and decode a mystery number. To start your mission, you have been granted access to the top secret files and here are the clues:



The sum of the digits is equal to 17.                      It is an odd number.  
The difference between the hundreds and ones digits is five less than the tens digit.

**MISSION:** Solve the following problems then write your answers on the Answers Column. Then, use the Cipher Table provided to find the corresponding code for each answer. Once you have all the codes, add them together to get the mystery number.

#### PROBLEMS

#### ANSWERS

#### CODES

1)  $\frac{1}{x} - \frac{1}{x^2} = \frac{4}{x}$

\_\_\_\_\_

2)  $\frac{4}{x} = \frac{12}{5x+10}$

\_\_\_\_\_

\* 3)  $\frac{1}{x+1} + \frac{1}{x^2-1} = \frac{2}{x^2-1}$

\_\_\_\_\_

4)  $\frac{1}{2} = \frac{1}{4} + \frac{x+3}{24x}$

\_\_\_\_\_

\* 5)  $\frac{2}{x-3} - \frac{1}{2x-6} = \frac{1}{2}$

\_\_\_\_\_

6)  $\frac{1}{5} + \frac{x+4}{x} = \frac{x-3}{x}$

\_\_\_\_\_

7)  $\frac{x}{x-2} - \frac{4}{x-1} = 1$

\_\_\_\_\_

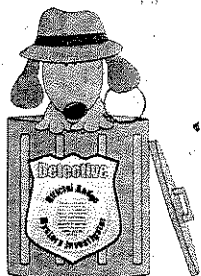
8)  $\frac{1}{3} - \frac{1}{6x} = \frac{2x-3}{3x}$

\_\_\_\_\_

CIPHER TABLE	
Answers	Codes
$\frac{3}{5}$	89
$-\frac{1}{3}$	76
5	54
-6	99
-35	79
-27	49
$\frac{5}{2}$	156
$\frac{1}{2}$	87
2	56
-5	57
$\frac{1}{3}$	62
3	88
7	59
6	64
$-\frac{2}{5}$	171

#### Mystery Number:

(Make sure that the mystery number satisfies all the given clues above)



Great Job, Agent(s)! You are certainly a great addition to our team. Keep your eyes open for your next mission and be ready to uncover the next mystery...