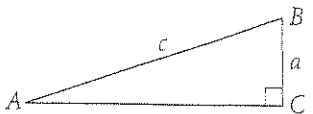
The Pythagorean Theorem and the Distance Formula

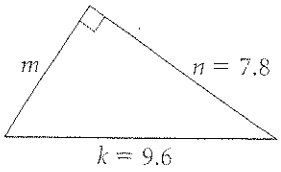
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. Use this relationship, known as the Pythagorean Theorem, to find the length of a side of a right triangle.

The Pythagorean Theorem



$a^2 + b^2 = c^2$

EXAMPLE 1 Find m in the triangle below, to the nearest tenth.



$$\begin{aligned} m^2 + n^2 &= k^2 \\ m^2 + 7.8^2 &= 9.6^2 \\ m^2 &= 9.6^2 - 7.8^2 \\ m^2 &= 31.32 \\ m &= \sqrt{31.32} \\ &\approx 5.6 \end{aligned}$$

Thus, m is about 5.6 units.

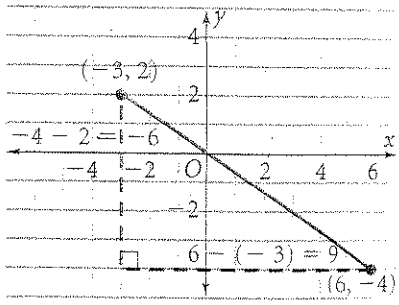
To find the distance between two points on the coordinate plane, use the distance formula.

The distance d between any two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

EXAMPLE 2 Find the distance between $(-3, 2)$ and $(6, -4)$.

$$\begin{aligned} d &= \sqrt{(6 - (-3))^2 + (-4 - 2)^2} \\ &= \sqrt{9^2 + (-6)^2} \\ &= \sqrt{81 + 36} \\ &= \sqrt{117} \\ &\approx 10.8 \end{aligned}$$

Thus, d is about 10.8 units.



EXERCISES

In each problem, a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse. Find each missing length.

1. c if $a = 6$ and $b = 8$
2. a if $b = 12$ and $c = 13$
3. b if $a = 8$ and $c = 17$
4. c if $a = 10$ and $b = 3$
5. a if $b = 7$ and $c = 25$
6. b if $a = 24$ and $c = 40$
7. a if $b = 100$ and $c = 114$
8. b if $a = 12.0$ and $c = 30.1$
9. c if $a = 8.3$ and $b = 3.3$

Find the distance between each pair of points, to the nearest tenth.

10. $(0, 0), (4, -3)$
11. $(-5, -5), (1, 3)$
12. $(-1, 0), (4, 12)$
13. $(0, 15), (17, 0)$
14. $(-4, 2), (4, -2)$
15. $(-8, -8), (8, 8)$
16. $(-1, 1), (1, -1)$
17. $(-2, 9), (0, 0)$
18. $(-5, 3), (4, 3)$
19. $(-2, 1), (3, 4)$
20. $(3, -2), (3, 5)$
21. $(5, 4), (-3, 1)$