Permutations

and

Combinations

Fundamental Counting Principle

Fundamental Counting Principle states that if an event has m possible outcomes and another independent event has n possible outcomes, then there are $m * n$ possible outcomes for the two events together.

Fundamental Counting Principle

Lets start with a simple example.

A student is to roll a die and flip a coin. How many possible outcomes will there be?

1H	2H	3H	4H	5H	6H
1T	2T	3T	4T	5T	6T

6*2 = 12 outcomes

12 outcomes

Fundamental Counting Principle

For a college interview, Robert has to choose what to wear from the following: 4 slacks, 3 shirts, 2 shoes and 5 ties. How many possible outfits does he have to choose from?

4*3*2*5 = 120 outfits

Permutations

A **Permutation** is an arrangement of items in a particular order.

Notice, **ORDER MATTERS**!

To find the number of Permutations of n items, we can use the Fundamental Counting Principle or factorial notation.

Permutations

The number of ways to arrange the letters ABC:

___ ___ ___

Number of choices for first blank? 3 ___ ___

Number of choices for second blank? 3 2 ___

Number of choices for third blank? 3 2 1

$3*2*1 = 6$ $3! = 3*2*1 = 6$

ABC ACB BAC BCA CAB CBA

Permutations

To find the number of Permutations of n items chosen r at a time, you can use the formula for finding P(n,r) or ${}_nP_r$

$${}_n p_r = \frac{n!}{(n-r)!} \quad \text{where } 0 \le r \le n.$$

$$\mathbf{{}_5 p_3 = \frac{5!}{(5-3)!}} = \frac{5!}{2!} = \mathbf{5 * 4 * 3 = 60}$$

Permutations

Practice:

A combination lock will open when the right choice of three numbers (from 1 to 30, inclusive) is selected. How many different lock combinations are possible assuming no number is repeated?

Answer Now

Permutations

Practice:

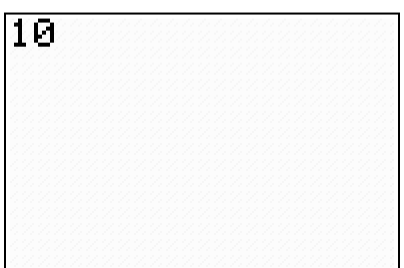
A combination lock will open when the right choice of three numbers (from 1 to 30, inclusive) is selected. How many different lock combinations are possible assuming no number is repeated?

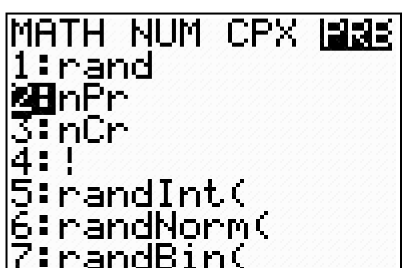
$$_{30}p_3 = \frac{\mathbf{30!}}{\mathbf{(30-3)!}} = \frac{30!}{27!} = \mathbf{30*29*28} = \mathbf{24360}$$

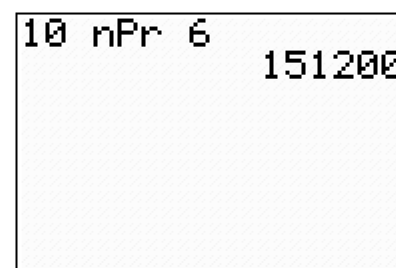
Permutations on the Calculator

You can use your calculator to find permutations

- To find the number of permutations of 10 items taken 6 at a time (${}_{10}P_6$):
- Type the total number of items
- Go to the MATH menu and arrow over to PRB
- Choose option 2: nPr
- Type the number of items you want to order







Permutations

Practice:

From a club of 24 members, a President, Vice President, Secretary, Treasurer and Historian are to be elected. In how many ways can the offices be filled?

Answer Now

Permutations

Practice:

From a club of 24 members, a President, Vice President, Secretary, Treasurer and Historian are to be elected. In how many ways can the offices be filled?

$$ {}_{24}p_5 = \frac{\mathbf{24!}}{\mathbf{(24-5)!}} = \frac{24!}{19!} = $$

$$ \mathbf{24 * 23 * 22 * 21 * 20 = 5{,}100{,}480} $$

Combinations

A **Combination** is an arrangement of items in which order does not matter.

ORDER DOES NOT MATTER!

Since the order does not matter in combinations, there are fewer combinations than permutations. The combinations are a "subset" of the permutations.

Combinations

To find the number of Combinations of n items chosen r at a time, you can use the formula

$$_{n}C_{r} = \frac{n!}{r!(n-r)!} \quad \text{where} \quad 0 \le r \le n.$$

Combinations

To find the number of Combinations of n items chosen r at a time, you can use the formula

$$_{n}C_{r} = \frac{n!}{r!(n-r)!} \quad \text{where } 0 \le r \le n.$$

$$_{5}C_{3} = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} =$$

$$\frac{5*4*3*2*1}{3*2*1*2*1} = \frac{5*4}{2*1} = \frac{20}{2} = 10$$

Combinations

Practice:

To play a particular card game, each player is dealt five cards from a standard deck of 52 cards. How many different hands are possible?

Answer Now

Combinations

Practice: To play a particular card game, each player is dealt five cards from a standard deck of 52 cards. How many different hands are possible?

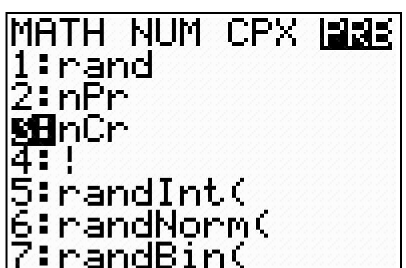
$$_{52}C_5 = \frac{52!}{5!(52-5)!} = \frac{52!}{5!47!} =$$

$$\frac{52*51*50*49*48}{5*4*3*2*1} = 2{,}598{,}960$$

Combinations on the Calculator

You can use your calculator to find combinations

- To find the number of combinations of 10 items taken 6 at a time (${}_{10}C_6$):
- Type the total number of items
- Go to the MATH menu and arrow over to PRB
- Choose option 3: nCr
- Type the number of items you want to order



Combinations

Practice:

A student must answer 3 out of 5 essay questions on a test. In how many different ways can the student select the questions?

Answer Now

Combinations

Practice: A student must answer 3 out of 5 essay questions on a test. In how many different ways can the student select the questions?

$$ {}_5C_3 = \frac{\mathbf{5!}}{\mathbf{3!(5-3)!}} = \frac{5!}{3!2!} = \frac{\mathbf{5*4}}{\mathbf{2*1}} = \mathbf{10} $$

Combinations

Practice:

A basketball team consists of two centers, five forwards, and four guards. In how many ways can the coach select a starting line up of one center, two forwards, and two guards?

Answer Now

Combinations

Practice: A basketball team consists of two centers, five forwards, and four guards. In how many ways can the coach select a starting line up of one center, two forwards, and two guards?

Center:

$$_2C_1 = \frac{2!}{1!1!} = 2$$

Forwards:

$$_5C_2 = \frac{5!}{2!3!} = \frac{5*4}{2*1} = 10$$

Guards:

$$_4C_2 = \frac{4!}{2!2!} = \frac{4*3}{2*1} = 6$$

$$_2C_1 * {}_5C_2 * {}_4C_2$$

Thus, the number of ways to select the starting line up is 2*10*6 = 120.

Probability with Permutations and Combinations

The 25-member senior class council is selecting officers for president, vice president and secretary. Emily would like to be president, David would like to be vice president, and Jenna would like to be secretary. If the offices are filled at random, beginning with president, what is the probability that they are selected for these offices?

Probability with Permutations and Combinations

- The students want specific offices so ORDER MATTERS!
- Find the total number of possible outcomes:
- $nPr = {}_{25}P_3 = 13800$
- Find the probability of that particular order:
- 1/13800 = a really really small percentage

Probability with Permutations and Combinations

The 25-member senior class council is selecting members for the prom committee. Stephen, Marcus and Sabrina want would like to be on this committee. If the members are selected at random, what is the probability that all three are selected for this committee?

Probability with Permutations and Combinations

- There are no specific positions/office so ORDER DOES NOT MATTER!
- Find the total number of possible outcomes:
- nCr = ${}_{25}C_3 = 2300$
- Find the probability of that combination of students:
- 1/2300 = a really really small percentage

Weird Examples

- Sometimes Permutation/Combination probability problems are A LOT less complicated when solved a different way.

Weird Examples

- Here is one of those examples:

  What is the probability that a randomly generated 4-letter arrangement of the letters in the word MONKEY ends with the letter K?

Weird Examples

- Think of choosing each letter as a series of events:
- Not K **AND** Not K **AND** Not K **AND** K
- Fill in the probabilities. Since we are finding the probability of all of these 4 events happening, we use the multiplication rule.

$$\boxed{\frac{5}{6}} \times \boxed{\frac{4}{5}} \times \boxed{\frac{3}{4}} \times \boxed{\frac{1}{3}} = \frac{1}{6}$$