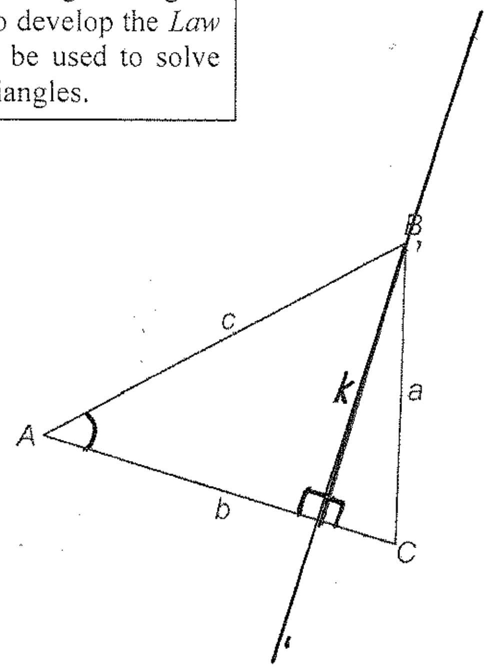
The Law of Sines

NAME Key

> Right triangle trigonometry can be used to solve problems involving right triangles. However, many interesting problems involve non-right triangles. In this lesson, you will use right triangle trigonometry to develop the *Law of Sines*. The law of sines is important because it can be used to solve problems involving non-right triangles as well as right triangles.



Consider oblique ΔABC shown to the right.

1. Sketch an altitude from vertex B.

2. Label the altitude k.

3. The altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles, and $\angle C$ is contained in the other. Using right triangle trigonometry, write two equations, one involving $\sin A$, and one involving $\sin C$.

   $\sin A = \frac{k}{c}$ $\qquad$ $\sin C = \frac{k}{a}$

4. Notice that each of the equations in Question 3 involves k. (Why does this happen?) Solve each equation for k.

   $c(\sin A) = \left(\frac{k}{\cancel{c}}\right)\cancel{c}$ $\qquad$ $k = a \sin C$

   $k = c \cdot \sin A$

5. Since both equations in Question 4 are equal to k, they can be set equal to each other. (Why is this possible?) Set the equations equal to each other to form a new equation.

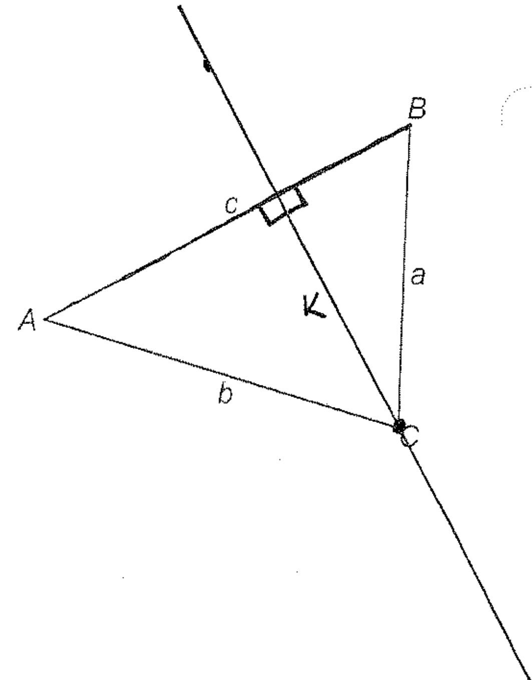
   $$c \sin A = a \sin C$$

6. Notice that the equation in Question 5 no longer involves k. (Why not?) Write an equation equivalent to the equation in Question 5, regrouping a with $\sin A$ and c with $\sin C$.

   $$\frac{\cancel{c}\sin A}{a\cancel{c}} = \frac{\cancel{a}\sin C}{\cancel{a}c}$$

   $$\frac{\sin A}{a} = \frac{\sin C}{c}$$

Again, consider oblique ΔABC.



7. This time, sketch an altitude from vertex C.

8. Label the altitude k.

9. The altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles and $\angle B$ is contained in the other. Using right triangle trigonometry, write two equations, one involving sin A and one involving sin B.

$\sin A = \frac{k}{b}$ $\qquad$ $\sin B = \frac{k}{a}$

10. Notice that each of the equations in Question 9 involves k. (Why does this happen?) Solve each equation for k.

$K = b\sin A$ $\qquad$ $k = a\sin B$

11. Since both equations in Question 10 are equal to k, they can be set equal to each other. (Why is this possible?) Set the equations equal to each other to form a new equation.

$$b\sin A = a\sin B$$

12. Notice that the equation in Question 11 no longer involves k. (Why not?) Write an equation equivalent to the equation in Question 11, regrouping a with sin A and b with sin B.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

13. Use the equations in Question 6 and Question 12 to write a third equation involving b, c, sin B, and sin C.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \frac{\sin(\text{Angle})}{\text{length of the opposite side}} = \frac{\sin(\text{Angle})}{\text{length of the opposite side}}$$

Together, the equations in Questions 6, 12, and 13 form the *Law of Sines*. The law of sines is important, because it can be used to solve problems involving both right and non-right triangles, because it involves only the sides and angles of a triangle.

Key

Unit 7 Notes: Law of Sines

I. Law of Sines

a. Law of Sines is used to find missing sides and angles in oblique triangles

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

b. When do you use Law of Sines? When you're given the following information:

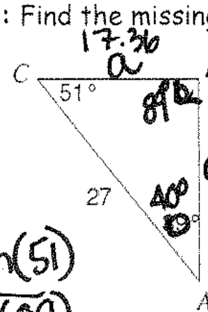
1. Angle-Angle-Side
2. Angle-Side-Angle

Ex:

3. Side-Side-Angle (SPECIAL CASE!!)

c. **Examples:** Find the missing sides and angles.

1.



$$\frac{\sin(89)}{27} = \frac{\sin(51)}{c}$$

$$\frac{c\sin(89)}{\sin(89)} = \frac{27\sin(51)}{\sin(89)}$$

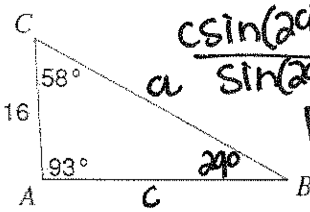
$$\boxed{c = 20.99}$$

$$\frac{\sin(89)}{27} = \frac{\sin(40)}{a}$$

$$\frac{a\cancel{\sin(89)}}{\cancel{\sin(89)}} = \frac{27\sin(40)}{\sin(89)}$$

$$\boxed{a = 17.36}$$

2.



$$\frac{\sin(29)}{16} = \frac{\sin(58)}{c}$$

$$\frac{c\sin(29)}{\sin(29)} = \frac{16\sin(58)}{\sin(29)}$$

$$\boxed{c = 27.999}$$

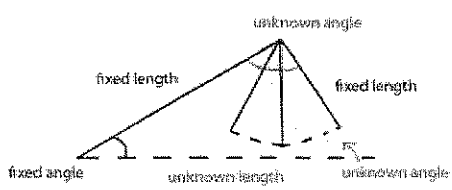
$$\frac{\sin(29)}{16} = \frac{\sin(93)}{a}$$

$$\frac{a\cancel{\sin(29)}}{\cancel{\sin(29)}} = \frac{16\sin(93)}{\sin(29)}$$

$$\boxed{a = 32.96}$$

Law of Sines-Ambiguous Case

a. Ambiguous Case is used when you are given side-side-angle

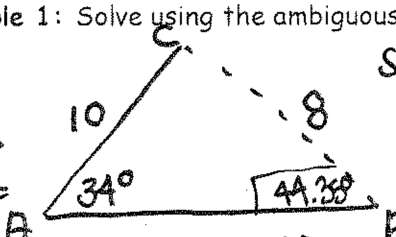


b. It is called ambiguous because we could have more than one answer!

c. Solving an ambiguous case:

1. Set up the problem and solve using Law of Sines
2. If you get one solution, always assume that there are two solutions.
3. Find the second possible solution
   FORMULA → 180° − solution1
4. Test the second angle
   TEST → given angle + solution 2 < 180°

Example 1: Solve using the ambiguous case



second ∡ = $\boxed{135.65}$

34 + 135.65 = 169.65

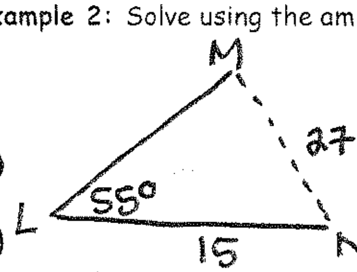
$$\frac{\sin(34)}{8} = \frac{\sin(B)}{10}$$

$$\frac{10\sin(34)}{8} = \frac{\cancel{8}\sin(B)}{\cancel{8}}$$

$$\sin(B) = \frac{10\sin(34)}{8}$$

$$B = \sin^{-1}\left(\frac{10\sin(34)}{8}\right)$$

Example 2: Solve using the ambiguous case



$$\frac{\sin(55)}{27} = \frac{\sin(M)}{15}$$

$$\frac{15\sin(55)}{27} = \frac{\cancel{27}\sin(M)}{\cancel{27}}$$

$$\boxed{M = 27.07°}$$

solution 2 = 180 − 27.07 = 152.93

55 + 152.93 = 207.93 ← DOESN'T WORK