Equation of a Circle

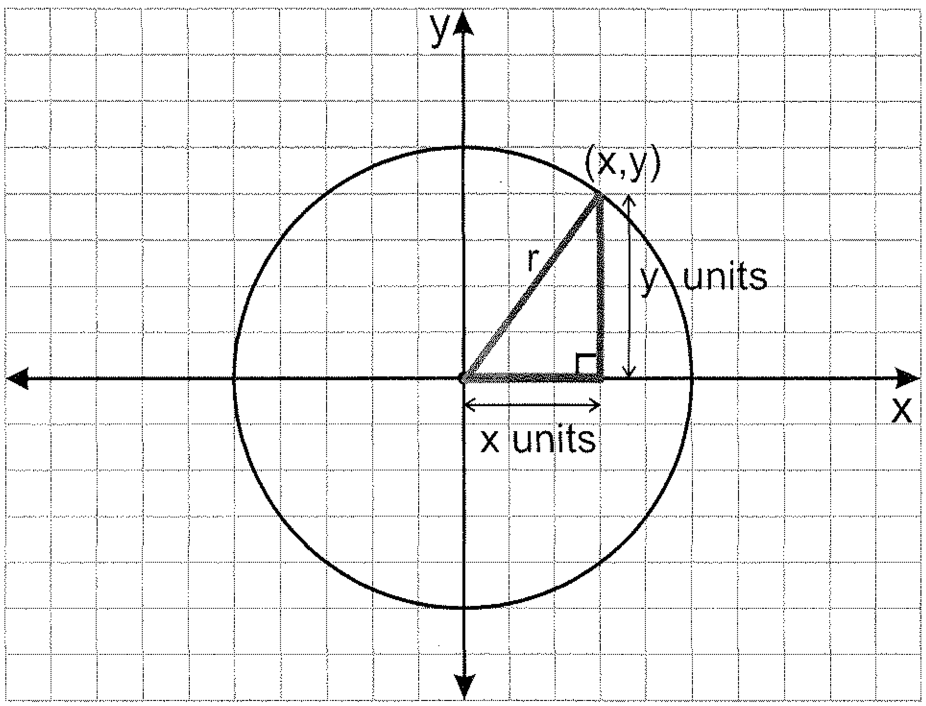
Name Answer Key ______ Period_____ Date______

The center of the circle below is located at the origin of the coordinate plane. We can use Pythagorean's Theorem to write an equation that relates the (x,y) coordinates that make up the circle with the circle's radius. If a right triangle is sketched inside of a circle with the acute vertices located at the circle's center and a point on the circle, then we are easily able to derive the equation for a circle.

Equation of circle below in terms of x, y, and r: $x^2 + y^2 = r^2$

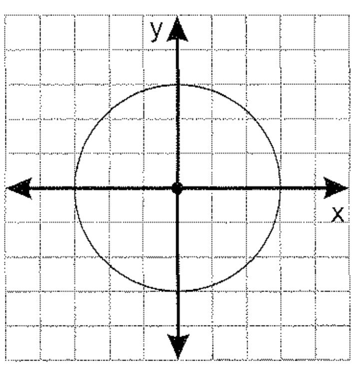
Now, count to find the radius for this example. Substitute this value for r in the equation you derived above.

Equation of circle below in terms of x and y: $x^2 + y^2 = 25$



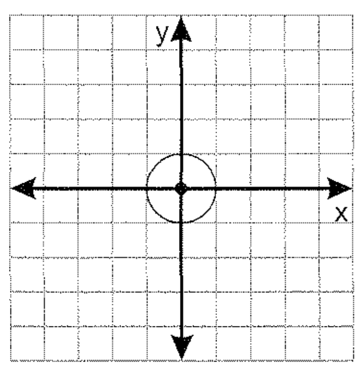
1) Practice: For each circle below, find the radius and then write the equation for the circle.

a)



$r = 3$; $x^2 + y^2 = 9$

b)

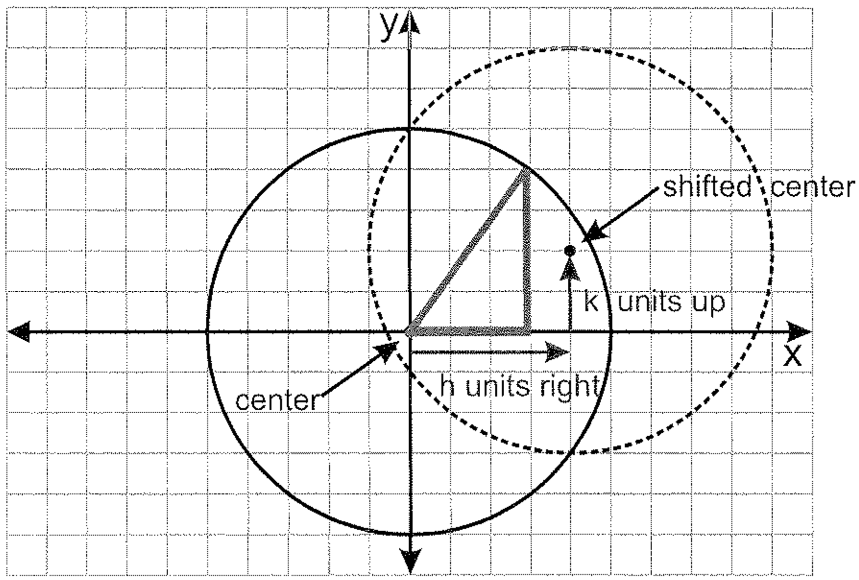


$r = 1$; $x^2 + y^2 = 1$

A translated circle will not be centered at the origin. The dashed circle below represents a translated circle.

Let h = horizontal shift
k = vertical shift

This shifts the translated circle's center to the coordinate (h,k).



Now, let's use the circle below to derive the equation for a circle not centered at the origin. We can still use Pythagorean's Theorem to complete this task.

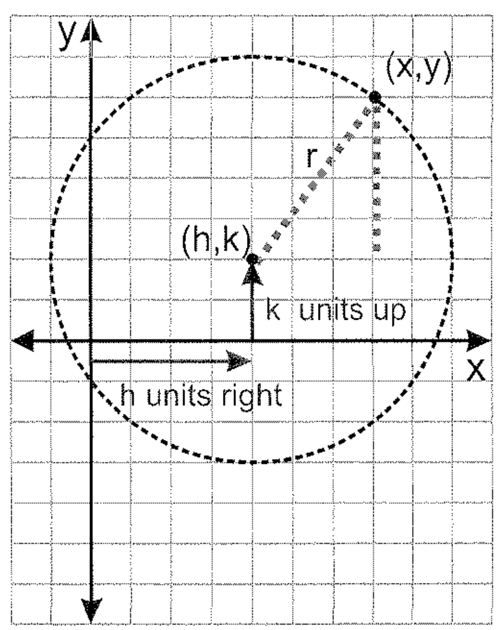
Find the length of each leg of the triangle:

horizontal leg = x-h vertical leg = y-k Center: (h, k)

Use these values in Pythagorean's Theorem to find the equation for the circle.

Equation of circle below in terms of x, y, and r:

$(x-h)^2 + (y-k)^2 = r^2$



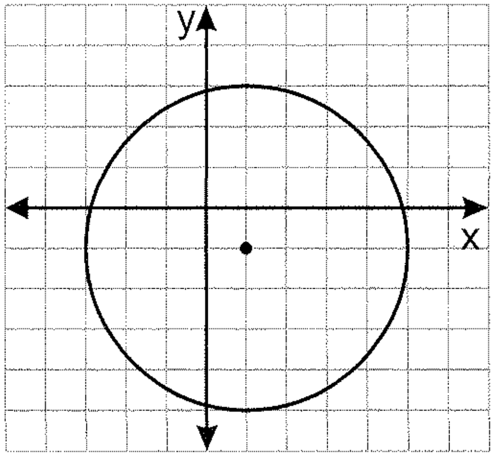
Now, count to find the radius for this example. Substitute this value for r in the equation you derived above. $r = 5$

Equation of circle below in terms of x and y:

$(x-4)^2 + (y-2)^2 = 25$

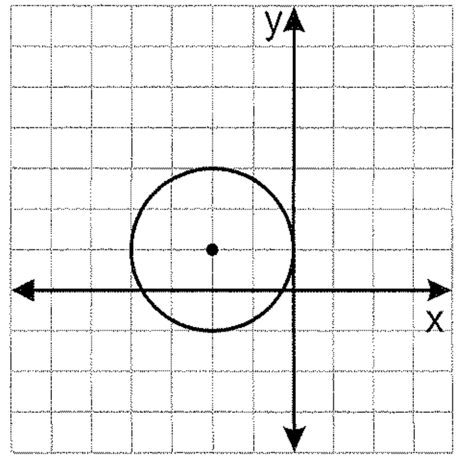
2) Practice: For each circle and its center graphed below, find the radius and then write the equation for the circle.

a)



$r = 4; (x-1)^2 + (y+1)^2 = 16$

b)



$r = 2; (x+2)^2 + (y-1)^2 = 4$

Graphing Circles: $(x-h)^2 + (y-k)^2 = r^2$

For the equation below, find the center and the radius. Then, graph the circle.

$(x-2)^2 + (y+3)^2 = 16$

To find h, think about what value is being subtracted from x: $(x- \underline{\ 2\ })$

To find k, think about what value is being subtracted from y: $(y- \underline{\ -3\ })$

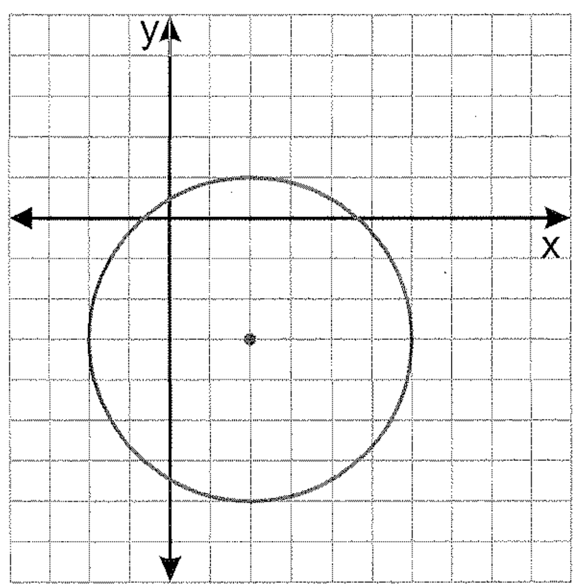
Next, find the radius by taking the square root of the right side of the equation, 16 for this example.

$r^2 = 16$, so $r = \sqrt{16} \longrightarrow r = 4$

Plot a point at the circle's center ($\underline{\ 2\ }$, $\underline{\ -3\ }$)

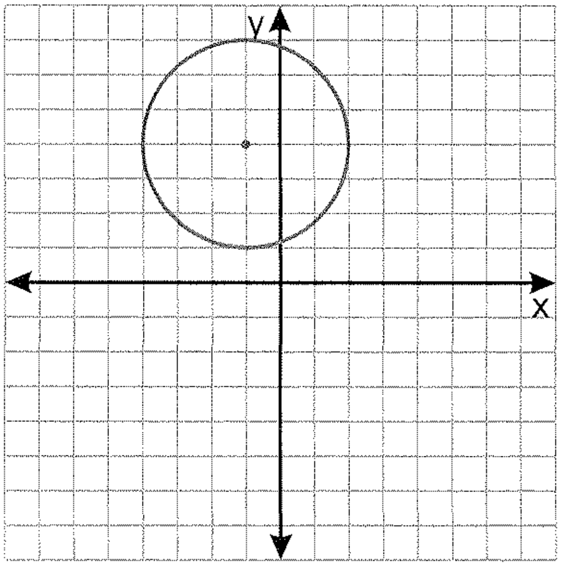
Use the radius to plot 4 more points from the center and sketch in the remainder of the circle.

Or, use a compass centered at the circle's center with the radius.

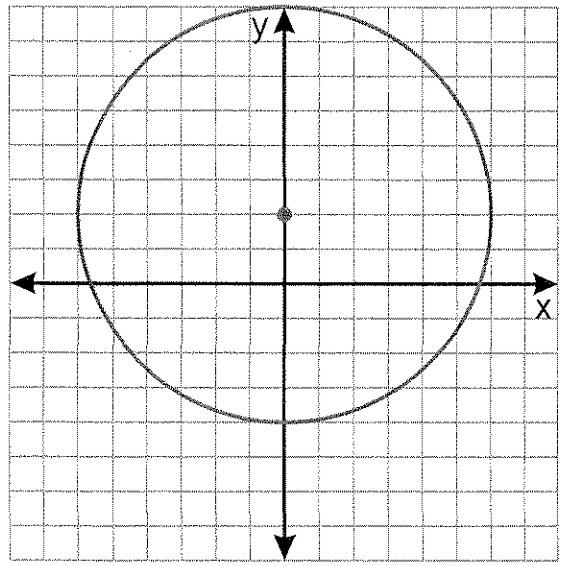


3) Practice: Graph the equation of each circle below.

a) $(x+1)^2 + (y-4)^2 = 9$



b) $(x)^2 + (y-2)^2 = 36$



Self Assess

★ Equation of a circle: $(x-h)^2 + (y-k)^2 = r^2$

Sketch a circle on the graph.

Write an equation for your circle:

$(x+2)^2 + (y-2)^2 = 9$

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