Graphing Exponential Functions + Unit 5 Review & Study Guide

Part I: Determine which functions are exponential functions. For those that are not, explain why they are not exponential functions.

(a) $f(x) = 2^x + 7$ Yes No

(b) $g(x) = x^2$ Yes No QUADRATIC

(c) $h(x) = 1^x$ Yes No

(d) $f(x) = x^x$ ~~Yes~~ No

(e) $h(x) = 3 \cdot 10^{-x}$ Yes No

(f) $f(x) = -3^{x+1} + 5$ Yes No

(g) $g(x) = (-3)^{x+1} + 5$ Yes No

(h) $h(x) = 2x - 1$ Yes No LINEAR

$y = a * b^x$

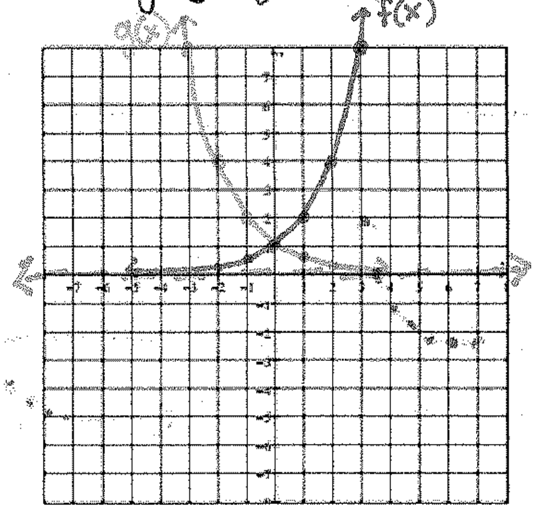
Part 2: Graph each of the following and find the domain and range for each function.

(a) $f(x) = 2^x$ domain: $\mathbb{R}$

range: $y > 0$

(b) $g(x) = \left(\frac{1}{2}\right)^x$ domain: $\mathbb{R}$

range: $y > 0$



Horizontal Asymptote: an "invisible" line that the graph of a function never crosses.

Identify the horizontal asymptote of (a):

$y = 0$

Identify the horizontal asymptote of (b):

$y = 0$

Identify the y-intercept of (a):

$(0, 1)$

Identify the y-intercept of (b):

$(0, 1)$

End behavior:

$x \to \infty$: increase

$x \to -\infty$: decrease

End Behavior:

$x \to \infty$: decrease

$x \to -\infty$: increase

Transforming Exponential Functions

$y = a * b^{x}$

Translate left or right: $g(x) = b^{x+c}$ (graph moves c units left)
$g(x) = b^{x-c}$ (graph moves c units right)

Vertical stretch or compression: $g(x) = cb^{x}$ (graph stretches if $c > 1$)
(graph shrinks if $0 < c < 1$)

Not one of our original transformations. Just be aware of what it is.

Horizontal stretch or compression: $g(x) = b^{cx}$ (graph shrinks if $c > 1$)
(graph stretches if $0 < c < 1$)

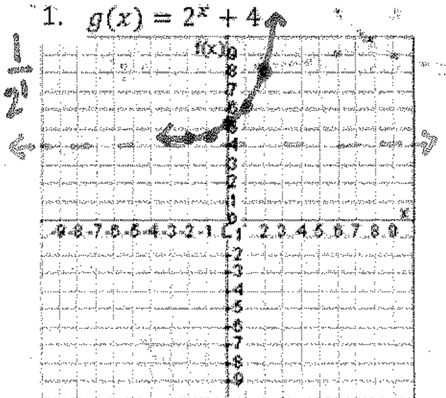
Reflections: $g(x) = -b^{x}$ (graph reflects over the x-axis)
$g(x) = b^{-x}$ (graph reflects over the y-axis)

Translate up or down: $g(x) = b^{x} + c$ (graph moves up c units)
$g(x) = b^{x} - c$ (graph moves down c units)
} moves the horizontal asymptote

Part 3: Describe the transformation using the function $f(x) = 2^x$ as the parent function. Then graph the function. For each, identify the domain, range, y-intercept, the asymptote, and the end behavior as x→ ∞ and -∞. horizontal asymptote.

1. $g(x) = 2^x + 4$

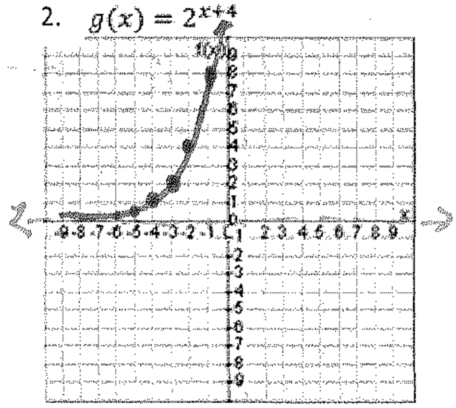
$2^{-1} = \frac{1}{2^1}$



translate up 4

Domain: $\mathbb{R}$
Range: $y > 4$
Y-Intercept: (0, 5)
Asymptote: $y = 4$
End Behavior: $x \to \infty$: ↑
$x \to -\infty$: ↓

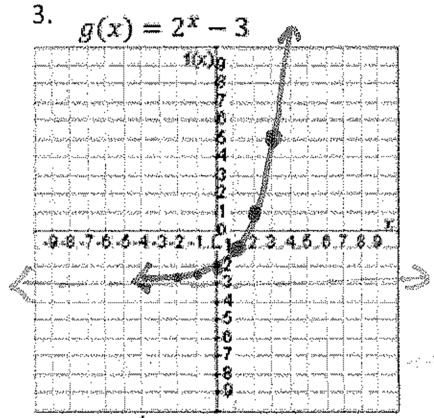
2. $g(x) = 2^{x+4}$



left 4

Domain: $\mathbb{R}$
Range: $y > \cancel{4}\ 0$
Y-Intercept: (0, 16)
Asymptote: $y = 0$
End Behavior: $x \to \infty$: ↑
$x \to -\infty$: ↓

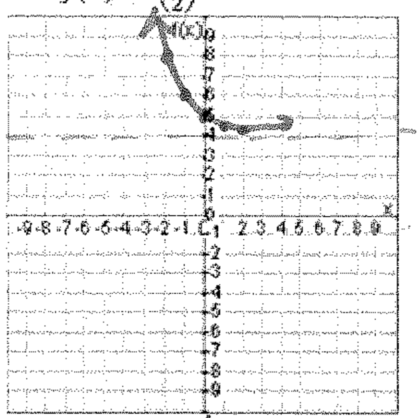
3. $g(x) = 2^x - 3$



down 3

Domain: $\mathbb{R}$
Range: $y > -3$
Y-Intercept: (0, -2)
Asymptote: $y = -3$
End Behavior: $x \to \infty$: ↑
$x \to -\infty$: ↓

4. $g(x) = \left(\frac{1}{2}\right)^{x} + 4$



up 4

Domain: $\mathbb{R}$

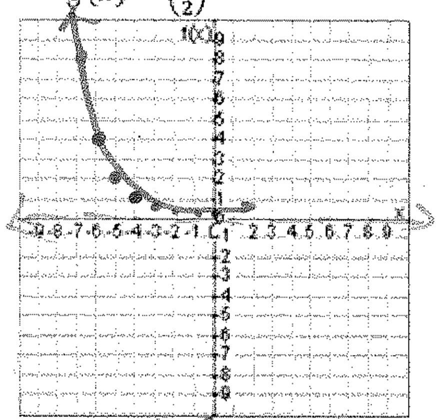
Range: $y > 4$

Y-Intercept: $(0, 5)$

Asymptote: $y = 4$

End Behavior: $x \to \infty: \downarrow$
$x \to -\infty: \uparrow$

5. $g(x) = \left(\frac{1}{2}\right)^{x+4}$



left 4

Domain: $\mathbb{R}$

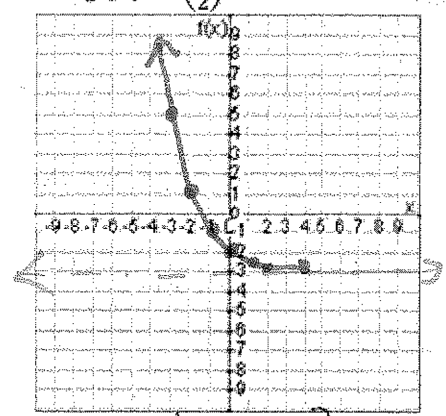
Range: $y > 0$

Y-Intercept: $(0, 1/16)$

Asymptote: $y = 0$

End Behavior: $x \to \infty: \downarrow$
$x \to -\infty: \uparrow$

6. $g(x) = \left(\frac{1}{2}\right)^{x} - 3$



down 3

Domain: $\mathbb{R}$

Range: $y > -3$

Y-Intercept: $(0, -2)$

Asymptote: $y = -3$

End Behavior: $x \to \infty: \downarrow$
$x \to \infty: \uparrow$

Review:

Part 5: Simplify the following by applying the properties of exponents.

1) $2m^2 \cdot 2m^3$

$4m^5$

2) $m^4 \cdot 2m^{-3}$

$2m$

3) $4r^{-3} \cdot 2r^2$

$\frac{8}{r}$

4) $4n^4 \cdot 2n^{-3}$

$8n$

5) $(2b^4)^{-1}$

$2^{-1}b^{-4} = \frac{1}{2b^4}$

6) $(x^2 y^{-1})^2$

$\frac{x^4}{y^2}$

7) $(2x^4 y^{-3})^{-1}$

$\frac{y^3}{2x^4}$

8) $(3m)^{-2}$

$\frac{1}{9m^2}$

9) $\dfrac{4x^0y^{-2}z^3}{4x}$

$\dfrac{z^3}{xy^2}$

10) $\dfrac{2h^3j^{-3}k^4}{3jk}$

$\dfrac{2h^3k^3}{3j^4}$

11) $\dfrac{4m^4n^3p^3}{3m^2n^2p^4} = \dfrac{4m^2n}{3p}$

12) $\dfrac{3x^3y^{-1}z^{-1}}{x^{-4}y^0z^0} = \dfrac{3x^7}{yz}$

Part 6: **Write each expression in radical form.**

1) $(5x)^{-\frac{5}{4}} = \dfrac{1}{\sqrt[4]{(5x)^5}}$

2) $(5x)^{-\frac{1}{2}} = \dfrac{1}{\sqrt{5x}}$

3) $(10n)^{\frac{3}{2}}$ $\sqrt{(10n)^3}$

4) $a^{\frac{6}{5}}$ $\sqrt[5]{a^6}$

Part 7: **Write each expression in exponential form.**

1) $(\sqrt[4]{m})^3$ $m^{\frac{3}{4}}$

2) $(\sqrt[3]{6x})^4$ $(6x)^{\frac{4}{3}}$

3) $\sqrt[4]{v}$ $v^{\frac{1}{4}}$

4) $\sqrt{6p}$ $(6p)^{\frac{1}{2}}$

Part 8: 1) Find the accumulated value of a \$5000 investment which is invested for 8 years at an interest rate of 12% compounded:

(a) annually

$y = 5000(1 + .12)^8 = \$12,379.82$

(b) semi-annually

$y = 5000\left(1 + \left(\frac{.12}{2}\right)\right)^{(8 \cdot 2)} = \$12,701.76$

(c) quarterly

$y = 5000\left(1 + \left(\frac{.12}{4}\right)\right)^{(8 \cdot 4)} = \$12,875.41$

(d) monthly

$y = 5000\left(1 + \left(\frac{.12}{12}\right)\right)^{(8 \cdot 12)} = \$12,996.36$

2) The exponential function $f(x) = 84.5(1.012)^x$ models the population of Mexico, $f(x)$, in millions, x years after 1986.

(a) Without using a calculator, substitute 0 for x and find Mexico's population in 1986. 84.5 million

(b) Estimate Mexico's population, to the nearest million in the year 2000.

$y = 84.5(1.012)$^14 $= 99.858$ million ≈ 100 mil.

(c) Estimate Mexico's population, to the nearest million, this year.

$y = 84.5(1.012)$^28 $= 118.007$ million ≈ 118 mil.

3) The half-life of radioactive carbon-14 is 5700 years. How much of an initial sample of 100mg will remain after 3000 years?

$y = 100 * \left(\frac{1}{2}\right)^{(3000/5700)} = 69.43$ mg

4) A customer purchases a television for \$800 using a credit card. The interest is charged on an unpaid balance at a rate of 18% per year compounded monthly. If the customer makes no payment for one year, how much is owed at the end of the year?

$y = 800\left(1 + \left(\frac{.18}{12}\right)\right)^{1 \cdot 12} = \$956.49$

5) A diamond ring was purchased twenty years ago for $500. The value of the ring increased by 8% each year. What is the value of the ring today?

$y = 500(1.08)^{20} = \$2{,}330.48$

6) A tool & die business purchased a piece of equipment of $250,000. The value of the equipment depreciates at a rate of 12% each year.

a. Write an exponential decay model for the value of equipment. $y = 250000(.88)^x$

b. What is the value of equipment after 5 years? $y = 250{,}000(.88)^5 = \$131{,}932.98$

c. Graph the model. ← On graph

d. Estimate when the equipment will have a value of $70,000 ~~10 years~~ $9.96 \approx$ 10 years

Helpful Hint: Count by 10,000 on your y-axis. Count by 2s on your x-axis.