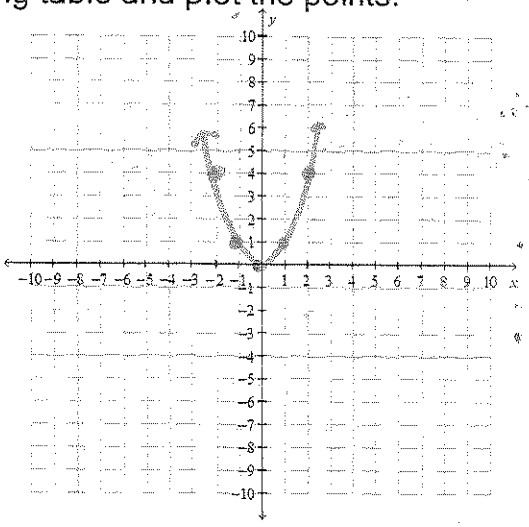
Graphing Quadratic Functions Name: _____
Unit 3 Day 2 Notes

Warm Up/Review:

The "parent" function for a quadratic function is $F(x) = x^2$. Its shape is a parabola.

Using this function, fill in the following table and plot the points.

x	F(x)
-2	4
-1	1
0	0
1	1
2	4



Below, describe the effect to the parent function for each of the following equations. Then, on the coordinate plane above, graph each of the equations below in a different color.

Remember your transformation rules from yesterday!

Equation	Effect on $F(x) = x^2$
1. $y = x^2 + 2$	TRANSLATE UP 2
2. $y = (x+1)^2$	TRANSLATE LEFT 1
3. $y = 2x^2$	STRETCH VERTICALLY BY 2
4. $y = -x^2$	REFLECT OVER X-AXIS

Lesson on Quadratics:

If we generalize all of our transformations into ONE equation, we get <u>VERTEX FORM</u> of a quadratic:

$$F(x) = a(x-h)^2 + k$$

h → moves the function left (if negative) or right (if positive)
*NOTE: two negatives create a positive, one positive and one negative create a negative

k → moves the function up (if positive) or down (if negative)

a → if a > 1: stretches the graph "a" amount away from the x-axis
if a < 1: compress the graph "a" amount towards the x-axis
if a is **negative** the graph is reflected over the x-axis

So what is the vertex? The **vertex of a quadratic function** is the minimum or maximum **point** on a quadratic function.

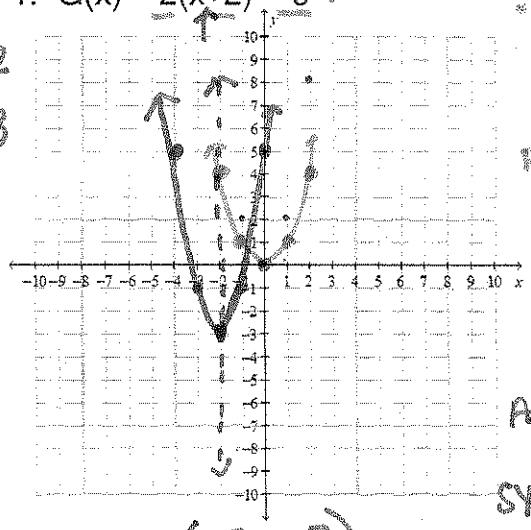
If the quadratic function opens upward, the vertex is a minimum.

If the quadratic function opens downward, the vertex is a maximum.

The **axis of symmetry** is a vertical **line** that goes through the vertex of a parabola and cuts the parabola in half.

Examples: Use your transformation rules to graph the following equations. Identify the vertex, tell whether it is a maximum or minimum, and draw in the line of symmetry.

1. $G(x) = 2(x+2)^2 - 3$

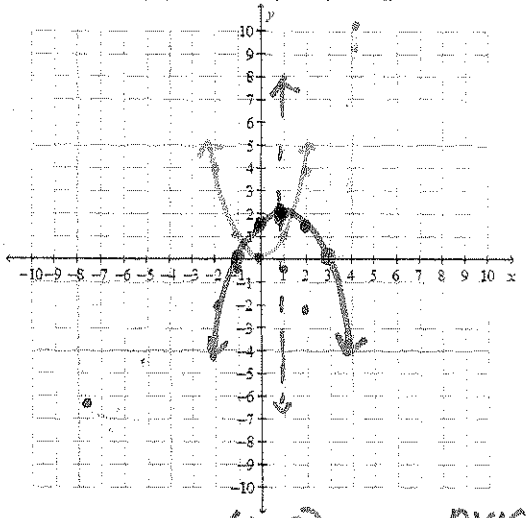


Vertex: $(-2, -3)$

Minimum or Maximum

Axis of Symmetry: $x = -2$

2. $H(x) = -1/2(x-1)^2 + 2$



Vertex: $(1, 2)$

Minimum or Maximum

Axis of Symmetry: $x = 1$

****NOTE:** You can find the vertex of an equation in vertex form WITHOUT graphing. The vertex is simply (h,k).

Examples: Without graphing, identify the vertex in the following equations:

Equation	Vertex
$F(x) = (x+5)^2 - 13$	$(-5, -13)$
$G(x) = -(x-4)^2 - 26$	$(4, -26)$
$H(x) = (x+100)^2 + 101$	$(-100, 101)$

The vertex form of a quadratic equation can sometimes be manipulated to get other forms. For example:

If we are given $y= -(x+2)^2 - 4$, note $(x+2)^2 = (x+2)(x+2)$

We can **FOIL/DISTRIBUTE** $(x+2)(x+2)$.

This gives us $x^2 + 4x + 4$

Therefore, $y= -(x^2+4x+4) - 4$

Distribute the negative → $y = -x^2 - 4x - 4 - 4$

Combine like terms → $y = -x^2 - 4x - 8$

This final form is called **STANDARD FORM**.

$$y = ax^2 + bx + c$$

While it is fairly easy to go from vertex form to standard form, it is not as easy to go from standard form to vertex form. You will learn how to do this in Math III. Because of this, it is also not as easy to graph standard form and identify the vertex. The following things will help:

1) You can graph/find the vertex in the calculator:
   a. Type your equation into y=
   b. Press GRAPH
   c. If you can't see your function, adjust your window
   d. Press 2nd TRACE (takes you to CALC)
   e. Choose 3 (if the vertex is minimum) or 4 (if the vertex is maximum)
   f. Scroll to the left of the vertex, hit ENTER
   g. Scroll to the right of the vertex, hit ENTER
   h. Press ENTER
   i. Your vertex will be shown at the bottom. $(-2, -4)$
2) You can use the following formulas:

Find the axis of symmetry first:

Use this formula:

$$x = \frac{-b}{2a}$$

Substitute the x-value back into the equation and solve for y. This is your vertex.

Examples: Find the axis of symmetry and vertex of the following equations:

1) $y = x^2 - 3x + 4$ $a=1$ $b=-3$ $c=4$

$x = \frac{-(-3)}{2(1)} = \frac{3}{2}$

$y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 4 = \frac{7}{4}$

$\left(\frac{3}{2}, \frac{7}{4}\right)$

2) $y = -2x^2+4x - 7$ a b c

$x = 1$

$y = -2(1)^2 + 4(1) - 7$

$(1, -5)$

$x = \frac{-4}{2(-2)} = \frac{-4}{-4}$

To graph standard form:

1) You can use the table in your calculator:
   a. Type your equation into y=
   b. Press 2nd GRAPH (takes you to TABLE)
   c. Scroll until you see where the numbers change direction (either from increasing to decreasing or from decreasing to increasing)
   d. Plot 5 points (include the vertex, two points to the left of the vertex, & two points to the right of the vertex)
2) Algebraically:
   a. Use your formula to find the axis of symmetry & the vertex and plot these.
   b. Make a table by substituting into your equation two x-values to the left of the vertex and two x-values to the right of the vertex. Plot these points.

Examples: Graph the following functions. Identify the vertex, y-intercepts, & x-intercepts.

1) $y = x^2 - 3x + 4$ $\quad a=1 \quad b=-3 \quad c=4$

x	y
0	4
1	2
→ 3/2	7/4
2	2
3	4

$x = \frac{-(-3)}{2(1)} = \frac{3}{2}$

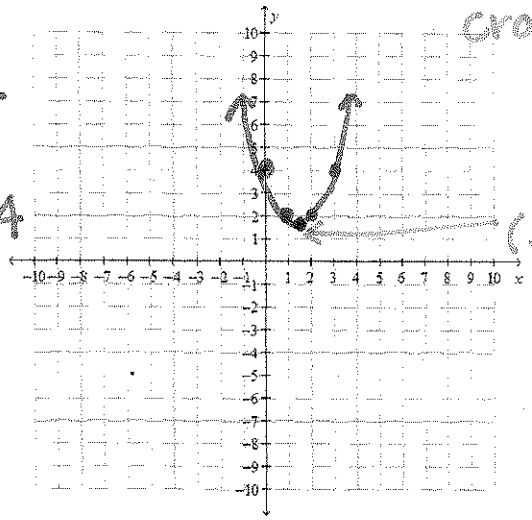
$y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 4$

$y = \frac{7}{4}$

↑ where the graph crosses the y-axis (0,4)

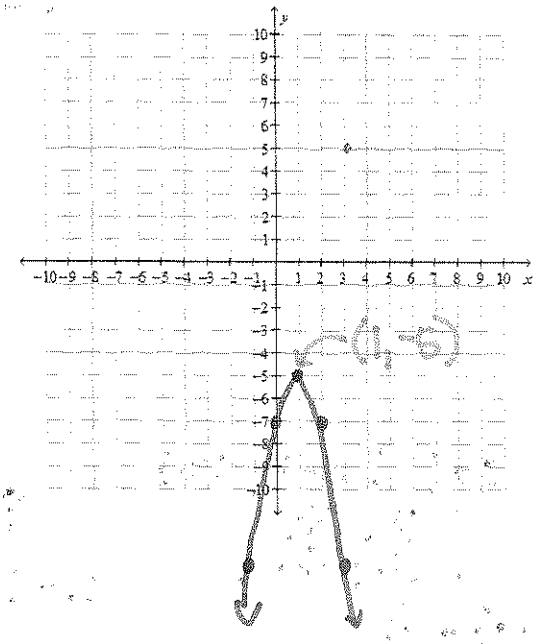
NO X-INTERCEPTS

(3/2, 7/4)



2) $y = -2x^2 + 4x - 7$

x	y
-1	-13
0	-7
1	-5
2	-7
3	-13



(1, -5)

Y-INT: (0, -7)

X-INT: NONE