CCM1B - UNIT 4: QUADRATIC FUNCTIONS

NAME: ______________________

Essential Questions

By the end of this unit, students will be able to answer the following questions.

- How do the coefficients determine the shape and the location of the graph of a quadratic function?
- What patterns of change are associated with quadratic functions?
- When is it appropriate to use a quadratic function to model the relationship between two quantities

Enduring Understandings

By the end of this unit, students will understand that . . .

- A second degree polynomial is a quadratic.
- Quadratic functions can be expressed in multiple forms.
- Factoring is a tool used to reveal the zeros of a quadratic function and zeros (also called x intercepts or roots) are solutions to the corresponding quadratic function.
- The rate of change of a quadratic function does not remain constant like a linear function and it does not continually increase (or decrease) like an exponential function. Rather, when considering equal intervals between points on the graph or in a table, the rate of change diminishes (grows) until the vertex of the graph is reached, and then grows (diminishes) after the vertex is reached.
- The graph of a quadratic function is in the shape of a parabola with a maximum or minimum representing the vertex.

Date	Day	Lesson	Assignment
THURS 3/7	1	CHARACTERISTICS OF QUADRATICS	
FRI 3/8	2	GRAPHING QUADRATIC FUNCTIONS	
MON 3/11	3	UNIT 4 QUIZ 1 SOLVING QUADRATICS BY FACTORING	
TUES 3/12	4	APPLICATIONS: AREA	
WED 3/13	5	APPLICATIONS: PROJECTILES	
THURS 3/14	6	UNIT 4 QUIZ 2 MORE PRACTICE W/ APPLICATIONS	
FRI 3/15	7	INTRO TO QUADRATIC REGRESSION	
MON 3/18	8	QUADRATIC REGRESSION CONTD.	
TUES 3/19	9	UNIT 4 REVIEW	
WED 3/20	10	UNIT 4 TEST	

Homework Grade:

Graphing Quadratic Functions

Quadratic Function: **standard form** ______________________ sometimes called a ___________

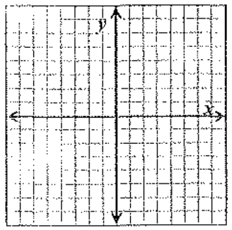
Vertex: ______________________

Axis of symmetry: ______________________

Form: $y = ax^2$. Graph each quadratic function. Label the vertex and axis of symmetry.

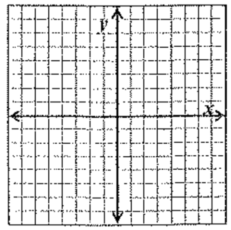
1. $y = x^2$

x	y
-2	
-1	
0	
1	
2	



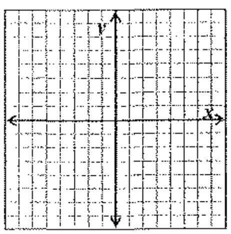
2. $y = -x^2$

x	y
-2	
-1	
0	
1	
2	



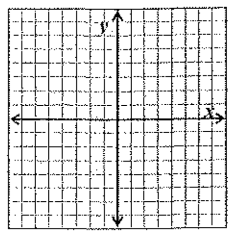
3. $y = 2x^2$

x	y
-2	
-1	
0	
1	
2	



4. $y = \frac{1}{3}x^2$

x	y
-6	
-3	
0	
3	
6	



4. Compare the graphs from #1 and #2. How are they similar? How do they differ?

5. Compare the graphs of #1, #3, and #4. How are they similar? How do they differ?

6. What is the y-intercept of each graph?

Graphing Quadratic Functions

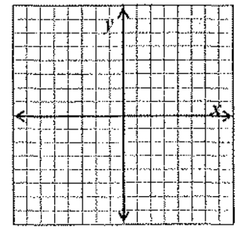
Based on Graphs #1 – 2, we can conclude that for $y = ax^2$:

- If $a > 0$, then the parabola will open ___________, the vertex will be ______ and the axis of symmetry will be _________.
- If $a < 0$, then the parabola will open ___________, the vertex will be ______ and the axis of symmetry will be _________.

Form: $y = ax^2 + c$.

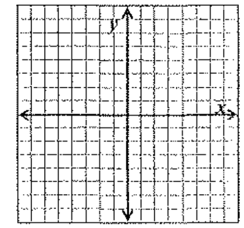
7. $y = x^2 + 1$

x	y
-2	
-1	
0	
1	
2	



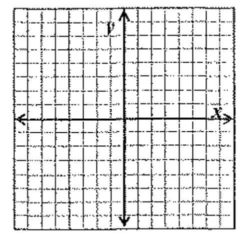
8. $y = x^2 - 2$

x	y
-2	
-1	
0	
1	
2	



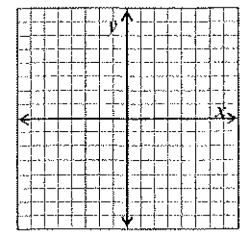
9. $y = -2x^2 - 3$

x	y
-2	
-1	
0	
1	
2	



10. $y = \frac{1}{3}x^2 + 2$

x	y
-6	
-3	
0	
3	
6	



11. Compare the graphs from #1, #7 and #8. How are they similar? How do they differ?

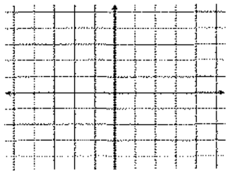
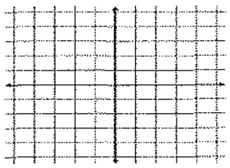
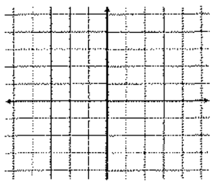
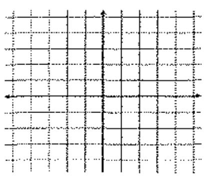
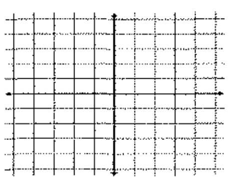
12. Compare the graphs from #3 and #9, then #4 and #10. How are they similar? How do they differ?

13. Find the y-intercept of #7 – 10. Compare the value of c and the y-intercept of each graph.

Based on Graphs #7 – 10, we can conclude that for $y = ax^2 + c$:

- The value of c determines the ____________________ of the graph.

2

Function	Sketch of Graph (from a calculator) *Remember to sketch the axis of symmetry!*	Range	Vertex	Axis of Symmetry	x-intercept	y-intercept	Increasing	Decreasing
$y = x^2 - 8x - 48$								
$y = x^2 - 16x + 55$								
$y = x^2 - 3x - 10$								
$y = x^2 - 3x - 28$								
$y = 2x^2 - 7x + 5$								

1) What is the relationship between the values of a, b, c and the y-intercept?

2) How are the vertex and the last two columns related to each other?

Warm up

Circle the correct answer. For the incorrect answer, explain why it is incorrect.

1. Which of the following is the vertex of $y = x^2 - 2x - 1$? For the incorrect answers, explain what part of the graph it is.

a. (1, -2) ____________ b. (0, -1) ____________

c. (2.41, 0) ____________ d. x=1 ____________

2. What is the axis of symmetry of $y = x^2 - 3x - 4$? For the incorrect answers, explain what that number represents in the equation.

a. x = 4 ____________ b. x = -1 ____________

c. x = -4 ____________ d. x = 1.5 ____________

3. What are the y-intercept of the function $y = 2x^2 + 4x + 3$? For the incorrect answers, explain why.

a. (-1, 1) ____________ b. (0, 3) ____________

c. (-2, 3) ____________ d. none ____________

4. What are the x-intercept(s) of the function $y = x^2 + 4x + 3$? If there is more than one, select both. For the incorrect answers explain why.

a. (0, -3) ____________ b. (0, -1) ____________

c. (3, 0) ____________ d. none ____________

5. Which graph is different from the others? Explain the major difference.

a. $y = x^2 + 3x + 2$ b. $y = -x^2 - 5x + 2$ c. $y = x^2 + 5x + 2$

Graphing Quadratics in Standard Form $y=ax^2+bx+c$

Steps for graphing a quadratic function:

Step 1: Find and graph the axis of symmetry and the vertex.

Step 2: Find and graph two other points on the graph. Use the y- intercept and x=o.

Step 3: Reflect the two points from step 2. Then draw the parabola.

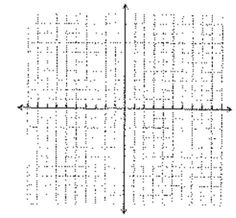
Graph the following using the above steps. Show work.

1. $y = -3x^2 + 6x + 5$

Step 1:

Step 2:

Step 3:

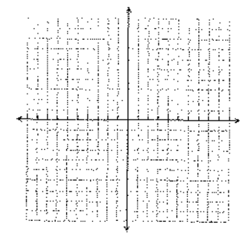


2. $y = x^2 + 4x$

Step 1:

Step 2:

Step 3

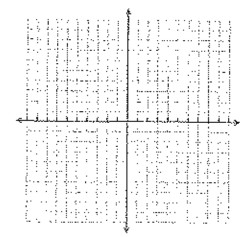


3. $f(x) = x^2 - 6$

Step 1:

Step 2:

Step:3



Factoring Review

Fill in the missing information.

Written in standard form	Written in factored form	Solve
$x^2+10x+9$		
$3x^2+9x$		
$2x^2-8$		
	$(x-7)(x+7)$	
$x^2-12x+27$		
$2x^2+5x+2$		
		$x=2,\ -9$
	$(5x+1)(x-3)$	
x^2-16		
	$(x+4)(x-6)$	
$7x^2+10x-8$		
$x^2-11x-60$		
		$x=-7,\ -3$
$10x^2+25x$		
$3x^2-2x-5$		

If the length of a field is $x-2$, and the total area is $5x^2-3x-14$, what is the width?

Find the area of a field that is $x+3$ meters long and $x+7$ meters wide.

Applications of Quadratics Notes

Name: ____________________

Helpful hints to solving word problems:

1.
2.
3.
4.

examples:

1. Verbal Representation

> You have a 1200-foot roll of fencing and a large field. You want to make two paddocks by splitting a rectangular enclosure in half. What are the dimensions of the largest such enclosure?

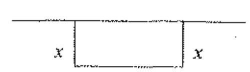
Draw a picture of the situation. Include variables in your picture.

Algebraic Representation ____________________

Answer using complete sentences ____________________

____

2. Bob is fencing in a pen along an outside wall of his house for his dog. Bob has 60 meters of fencing and wants to give his dog the greatest possible area. Find the dimensions of the pen.



3. If each of the sides of a square is lengthened by 6 cm, the area becomes 144 cm^2. Find the length of one side of the original square.

4. Steve is planning a garden that is 25 m longer than it is wide. The garden will have an area of 7500 m^2. What will its dimensions be?

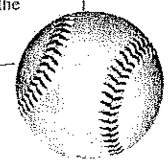
5. A flower bed is to be 3 m longer than it is wide. The flower bed will have an area of 108 m^2. What will its dimensions be?

6. The length of a pool at a local YMCA is 10 feet more than its width. A walkway 4 feet wide surrounds the outside of the pool. If the total area of the walkway and pool is 999 square feet, find the dimensions of the pool.

Projectile Motion Notes   Name____________

1. Verbal Representation

A baseball is "popped" straight up by a batter with an initial velocity of 64 ft/sec. The height of the ball above ground is given by the function $y = -16t^2 + 64t + 3$, where t is time in seconds after the ball leaves the bat and y is the height in feet. Although the path of the ball is straight up and down, the graph of its height as a function of time is a parabola. The ball goes up fast at first and then more slowly because of gravity. What is the maximum height that the ball reaches? When does it reach its maximum height? When will the ball hit the ground?



Projectile motion starts with the same formula:

$y = \frac{1}{2}gt^2 + v_o t + h_o$ where v_o is the initial velocity, h_o is the initial height, and g is acceleration due to gravity. The gravity constant will depend on what types of units are in the problem – feet or meters. $g = -9.8\ m/s^s$ or $-32\ ft/s^2$.

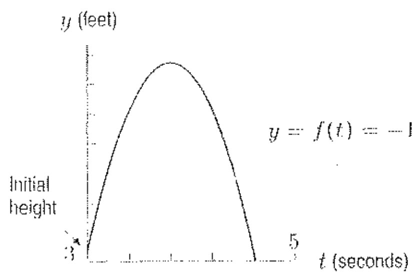
Why is g negative?____________

Now, back to the problem...

Algebraic representation ____________

Graphic representation

Problem adapted from *Functions Modeling Change* by Connally, Hughes-Hallett, and Gleason



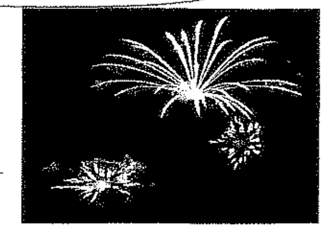
Answer the questions using complete sentences:____________

____

____

2. Verbal Representation:

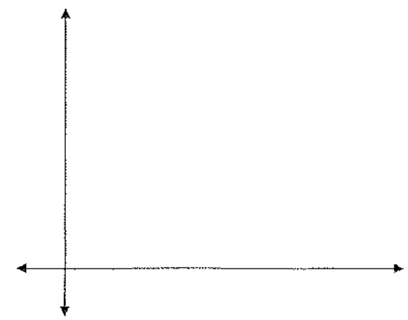
Suppose a particular star is projected from an aerial firework at a starting height of 520 feet with an initial upward velocity of 72 ft/s. The equation $h = -16t^2 + 72t + 520$ gives the star's height h in feet at time t in seconds. How long will it take for the star to reach maximum height? How far above the ground will it be?



Algebraic Representation

____

Graphic Representation



Answer the questions using complete sentences:____________

____

____

Applications of Quadratics Practice Name____________________

1. From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 32 feet per second. The height of the ball t seconds after Susan throws it is given by $h(t) = -16t^2 + 32t + 4$.

a. Find the maximum height reached by the ball and the time this height is reached.
b. When was the ball at the same height as when it was thrown?

2. Marta throws a baseball with an initial upward velocity of 70 feet per second. This equation $h(t) = -16t^2 + 70t$ models the situation.

a. Ignoring Marta's height, how long after she releases the ball, will it hit the ground?
b. What is the maximum height of the baseball?

3. A volcanic eruption blasts a boulder upward with an initial velocity of 240 feet per second. This is modeled by the equation $h(t) = -16t^2 + 240t$.

a. How long will it take the boulder to hit the ground?
b. How high was the boulder after 5 seconds?

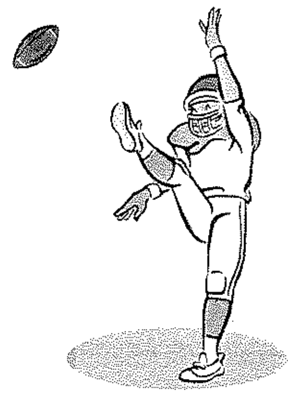
4. A baseball player hits a high pop-up with an initial upward velocity of 30 meters per second, 1.4 meters above the ground. The height of the ball (in meters) t seconds after being hit is modeled by $h(t) = -4.9t^2 + 30t + 1.4$.

a. How long will it take for the baseball to hit the ground?
b. What time will the ball be 15 meters high?

5. A rectangular lot is 50 feet wide and 60 feet long. If both the width and the length are increased by the same amount, the area is increased by 1200 square feet. Find the amount by which both the width and the length are increased.

6. A rectangular lawn is 60 feet by 80 feet. How wide of a uniform strip must be cut around the edge when mowing the grass in order for half of the lawn to be cut?

Real World Connections **Name:_______________________**



1. **Are You Ready For Some Football?** The height of a punted football can be modeled with the quadratic function $h = -0.01x^2 + 1.18x + 2$. The horizontal distance in feet from the point of impact with the kicker's foot is x, and h is the height of the ball in feet.
   - **a)** What is the ball's height when it has traveled 30 ft downfield?
   - **b)** What is the maximum height of the punt? How far downfield has the ball traveled when it reaches its maximum height?
   - **c)** The nearest defensive player is 5 ft horizontally from the point of impact. How high must he get his hand to block the punt?
   - **d)** Suppose the punt was not blocked but continued on its path. How far down field would the ball go before it hit the ground?
   - **e)** Why is the linear equation $h = 1.13x + 2$ not a good model for the path of the football? Explain.

2. **More Football** Although a football field appears to be flat, its surface is actually shaped like a parabola so that rain runs off to either side. The cross section of a field with synthetic turf can be modeled by $y = -0.000234(x-80)^2 + 1.5$ where x and y are measured in feet.
   - **a)** What is the field's width?
   - **b)** What is the maximum height of the field's surface?

Source: Boston College

3. **Newspaper Circulation** The function $f(x) = -0.019x^2 + 3.04x - 58.87$ describes newspaper circulation (in millions) in the United States for 1920 to 1998 (where $x = 20$ is used for 1920). Identify periods of increasing and decreasing circulation.
   - **a)** According to the function, when did newspaper circulation peak?
   - **b)** When will circulation approximate 45 million?

4. **Civil Engineering** The Golden Gate Bridge in San Francisco has two towers that rise 500 feet above the road and are connected by suspension cables as shown. Each cable forms a parabola with equation $y = \frac{1}{8960}(x-2100)^2 + 8$ where x and y are measured in feet.
   - **a)** What is the distance between the two towers?
   - **b)** What is the height above the road of a cable at its lowest point?

Modeling Data With Quadratic Functions

1. A study compared the speed x (in miles per hour) and the average fuel economy y (in miles per gallon) for cars.

Speed, x	15	20	25	30	35	40
Fuel, y	22.3	25.5	27.5	29.0	28.8	30.0
Speed, x	45	50	55	60	65	70
Fuel, y	29.9	30.2	30.4	28.8	27.4	25.3

a. Find a quadratic model for the data.

b. Find the speed that maximizes a car's fuel economy.

2. The table below shows the height of a column of water as it drains from its container.

Elapsed Time (seconds)	0	10	20	30	40	50	60
Water Level (millimeters)	120	100	83	66	50	37	28

a. Model the data with a quadratic function.

b. Use the model to estimate the water level at 35 seconds.

3. A man throws a ball off the top of a building. The table shows the height of the ball at different times.

a. Find a quadratic function to model the data.

b. Use the model to estimate the height of the ball at 2.5 seconds.

Time (seconds)	0	1	2	3
Height (feet)	46	63	48	1

4. The table shows the time t it takes to boil a potato whose smallest diameter (that is, whose shortest distance through the center) is d.

Diameter (mm), d	20	25	30	35	40	45	50
Boiling Time (min), t	27	42	61	83	109	138	170

a. Find a linear model. Graph the model.

b. Find a quadratic model. Graph the model.

c. Which model fits best?

d. Use the best model to find the time needed to boil a potato whose diameter is 55 mm.

Data analysis
What is the best line to use?

Name: ____________________
Date : ____________________

There are 3 types of models that we have used to model data:

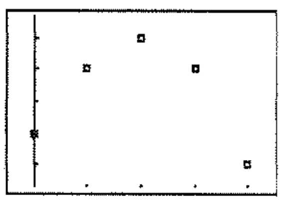
Linear: $y = mx + b$

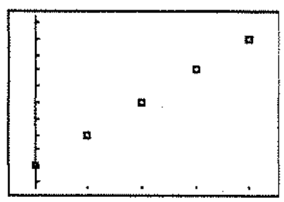
Quadratic: $y = ax^2 + bx + c$

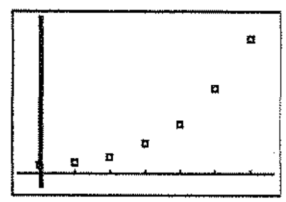
Exponential: $y = a(b)^x$

Each of these models can be used on any set of data. However, the "Best" model is the one that goes through as much of the data points as possible and yields the best predictions.

To determine which model to use we first plot the data and look at the shape it makes. Which model would be best for the following scatter plots?







________________ ________________ ________________

The table shows the growth of a certain bacteria

Time in Hours,x	0	1	2	3	4	5
Number of Cells, N	50	71	100	141	200	283

1. Put the data into your stat editor and plot it. Which model looks like it would be best?

2. Find each model and determine which one fits the data the best:
   Linear: Go to stat → Calc → LinReg → Enter
   After LinReg Type: L1, L2, Y1 and then hit enter
   Quadratic: Go to stat → Calc → QuadReg → Enter
   After QuadReg Type: L1, L2, Y2 and then hit enter
   Exponential: Go to stat → Calc → ExpReg → Enter
   After ExpReg Type: L1, L2, Y3 and then hit enter
3. Hit graph and you will see each model graphed in order. As they are graphed, observe which one goes through the most data points. This is the best model. Which model did the calculator find to be the best?

UNIT 4: QUADRATIC FUNCTIONS

DATE	LESSON	HOMEWORK
WED 3/6	CHARACTERISTICS	
THURS3/7	GRAPHING	
FRI3/8	**QUIZ**/SOLVING	
MON 3/11	APPLICATIONS: AREA	
TUES 3/12	APPLICATIONS: PROJECTILES	
WED 3/13	APPLICATIONS: MIXED	
THURS 3/14	REVIEW/**QUIZ**	
FRI 3/15	INTRO TO REGRESSION	
MON 3/18	REGRESSION	
TUES 3/19	UNIT 4 REVIEW	
WED 3/20	**UNIT 4 TEST**	

OBJECTIVES:

F-IF.4	FOR A FUNCTION THAT MODELS A RELATIONSHIP BETWEEN TWO QUANTITIES, INTERPRET KEY FEATURES OF GRAPHS AND TABLES IN TERMS OF THE QUANTITIES, AND SKETCH GRAPHS SHOWING KEY FEATURES GIVEN A VERBAL DESCRIPTION OF THE RELATIONSHIP.
F-IF.7	GRAPH FUNCTIONS EXPRESSED SYMBOLICALLY AND SHOW KEY FEATURES OF THE GRAPH, BY HAND IN SIMPLE CASES AND USING TECHNOLOGY FOR MORE COMPLICATED CASES. A. GRAPH LINEAR AND QUADRATIC FUNCTIONS AND SHOW INTERCEPTS, MAXIMA, AND MINIMA.
A-CED.2	CREATE EQUATIONS IN TWO OR MORE VARIABLES TO REPRESENT RELATIONSHIPS BETWEEN QUANTITIES; GRAPH EQUATIONS ON COORDINATE AXES WITH LABELS AND SCALES.
A-SSE.3	CHOOSE AND PRODUCE AN EQUIVALENT FORM OF AN EXPRESSION TO REVEAL AND EXPLAIN PROPERTIES OF THE QUANTITY REPRESENTED BY THE EXPRESSION. A. FACTOR A QUADRATIC EXPRESSION TO REVEAL THE ZEROS OF THE FUNCTION IT DEFINES (FACTORING TIED TO A SPECIFIC PURPOSE, NOT AS A SEPARATE UNIT).
F-IF.8	WRITE A FUNCTION DEFINED BY AN EXPRESSION IN DIFFERENT BUT EQUIVALENT FORMS TO REVEAL AND EXPLAIN DIFFERENT PROPERTIES OF THE FUNCTION. A. USE THE PROCESS OF FACTORING AND COMPLETING THE SQUARE IN A QUADRATIC FUNCTION TO SHOW ZEROS, EXTREME VALUES, AND SYMMETRY OF THE GRAPH, AND INTERPRET THESE IN TERMS OF A CONTEXT.

Name: ______________________________

Quadratic Equations

For each problem, find the a) vertex, b) axis of symmetry, c) x-intercepts, and d)y-intercepts.

1. $1x^2 - 9x + 18 = 0$
x = **3 and 6**

2. $1x^2 - 10x + 9 = 0$
x = **1 and 9**

3. $1x^2 - 10x + 16 = 0$
x = **2 and 8**

4. $-1x^2 - 5x + 6 = 0$
x = **1 and -6**

5. $-1x^2 + 11x - 24 = 0$
x = **8 and 3**

6. $1x^2 - 1x - 2 = 0$
x = **-1 and 2**

7. $-1x^2 - 1x + 12 = 0$
x = **3 and -4**

8. $-1x^2 + 6x + 7 = 0$
x = **7 and -1**

9. $-1x^2 + 10x - 16 = 0$
x = **8 and 2**

10. $-1x^2 - 2x + 35 = 0$
x = **5 and -7**

11. $1x^2 + 1x - 12 = 0$
x = **-4 and 3**

12. $1x^2 - 7x + 10 = 0$
x = **2 and 5**

13. $-1x^2 + 6x + 16 = 0$
x = **8 and -2**

14. $-1x^2 - 4x + 32 = 0$
x = **4 and -8**

15. $1x^2 - 12x + 35 = 0$
x = **5 and 7**

16. $1x^2 - 10x + 16 = 0$
x = **2 and 8**

17. $-1x^2 + 14x - 45 = 0$
x = **9 and 5**

18. $-1x^2 - 1x + 56 = 0$
x = **7 and -8**

19. $1x^2 - 14x + 48 = 0$
x = **6 and 8**

20. $1x^2 + 0x - 9 = 0$
x = **-3 and 3**

21. $-1x^2 - 2x + 35 = 0$
x = **5 and -7**

22. $1x^2 + 12x + 27 = 0$
x = **-9 and -3**

23. $1x^2 - 3x - 4 = 0$
x = **-1 and 4**

24. $-1x^2 - 6x + 7 = 0$
x = **1 and -7**

25. $1x^2 - 4x - 32 = 0$
x = **-4 and 8**

26. $-1x^2 - 14x - 45 = 0$
x = **-5 and -9**

27. $-1x^2 - 10x - 9 = 0$
x = **-1 and -9**

28. $1x^2 + 13x + 42 = 0$
x = **-7 and -6**

29. $1x^2 - 7x - 8 = 0$
x = **-1 and 8**

30. $1x^2 - 2x + 1 = 0$
x = **1 and 1**