CCM1B - Unit 2: Exponential Functions Applications Name: ____________________

Objectives:

Essential Questions

By the end of this unit, students will be able to answer the following questions:

1. What considerations should be taken into account when determining the boundaries and scales of a graph?
2. What are the key features of an exponential function?
3. When given one of the four forms of information, what should be taken into consideration when determining the best function to model the situation? Explain.
4. Why is a multiplicative rate of change the key feature of an exponential function and how is it revealed in the different forms of this function (verbal, graph, table, equation)?
5. How do you determine the best model for a data pattern?
6. When given a sequence, how do I identify whether it is arithmetic or geometric and how do I write a rule for the sequence?

Enduring Understandings

By the end of the unit, the student will understand that:

1. The context of a problem determines the domain and range as well as an appropriate scale of an exponential function.
2. The equation of an exponential function reveals the initial value and the rate of change for a given situation.
3. The points on the graph of the equation represent the solutions to exponential relationships.
4. The exponential relationship between two quantities can be expressed graphically and with a symbolic rule.
5. Exponential functions model real world problems, of growth and decay, such as monetary growth, population increases or decreases, car values, half-life, etc.
6. Linear and exponential functions exhibit different characteristics.
7. One type of function does not fit all situations in life.

Exponential functions can be written as explicit expressions or using the recursive process.

Date	Day	Lesson	Assignment
Thurs. 2/7	1	Drug Filtering Lab Depreciation Problems	
Fri. 2/8	2	Half Life	
Mon. 2/11	3	**Unit 2 Quiz:** Depreciation & Half-Life Problems	
Tues 2/12	4	Compound Interest	
Wed. 2/13	5	Growth Problems	
Thurs. 2/14	6	Unit 2 Review	
Fri. 2/15	7	**Unit 2 Test** Early Release	

Homework Grade: [ ]

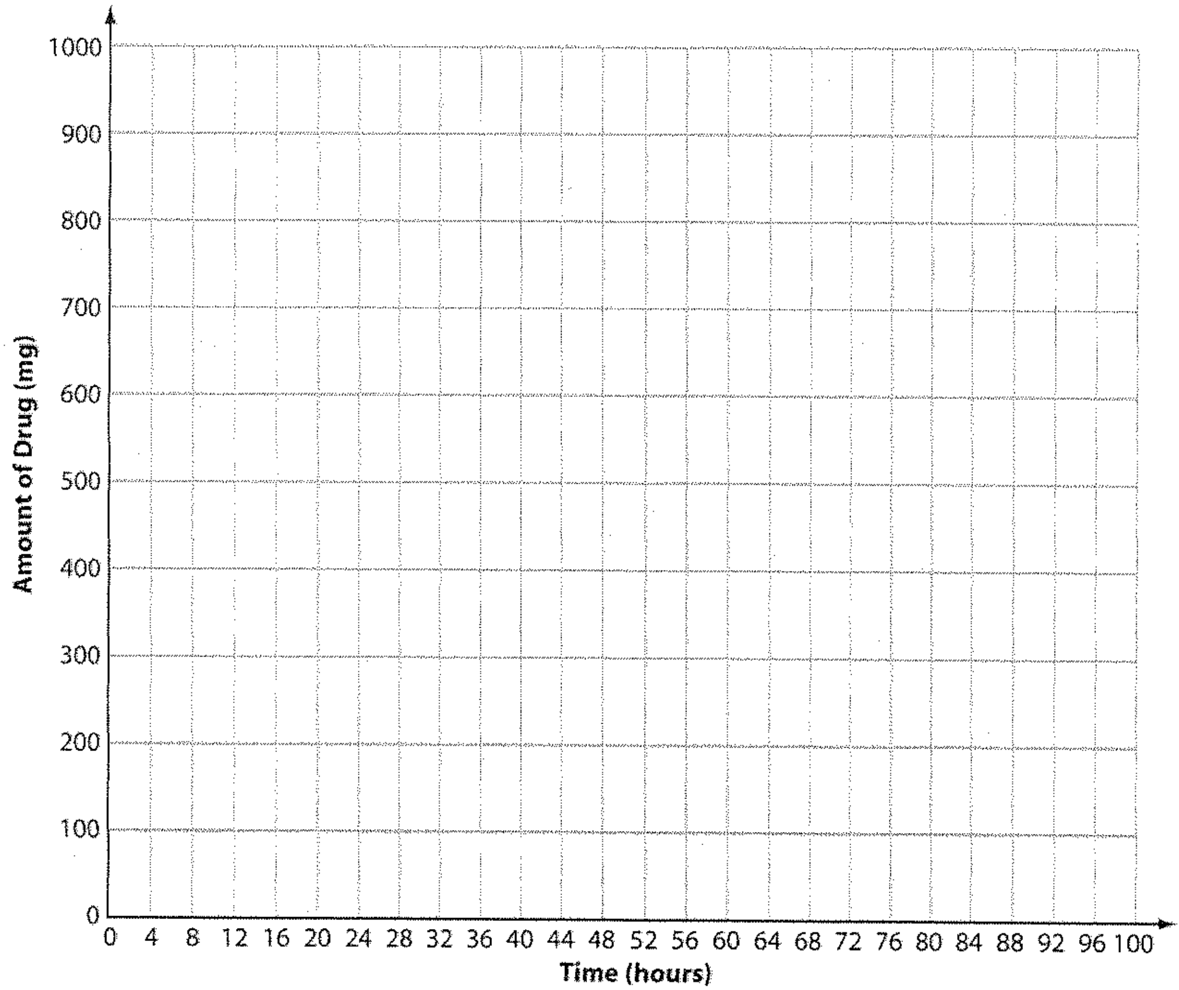
Drug Filtering

NAME______________________________

1. Assume that your kidneys can filter out 25% of a drug in your blood every 4 hours. You take one 1000-milligram dose of the drug. Fill in the table showing the amount of the drug in your blood as a function of time. The first two data points are already completed. Round each value to the nearest milligram.

TIME SINCE TAKING THE DRUG (HR)	AMOUNT OF DRUG IN YOUR BLOOD (MG)
0	1000
4	750
8	
12	
16	
20	
24	
28	
32	
36	
40	
44	
48	
52	
56	
60	
64	
68	

2. Graph the data below.



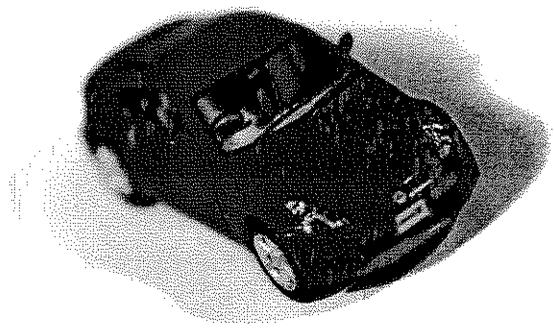
3. How many milligrams of the drug are in your blood after 2 days?

4. Will you ever completely remove the drug from your system? Explain your reasoning.

5. A blood test is able to detect the presence of the drug if there is at least 0.1 mg in your blood. How many days will it take before the test will come back negative? Explain your answer.

Day 1 Notes

Exponential Decay: Depreciation Problems



Most cars lose value each year by a process known as **depreciation.** You may have heard before that a new car loses a large part of its value in the first 2 or 3 years and continues to lose its value, but more gradually, over time. That is because the car does not lose the same **amount** of value each year, it loses approximately the same **percentage** of its value each year. What kind of model would be useful for calculating the value of a car over time?

Let us look at an example of depreciation: Suppose the value of car when new is $20,000 and it depreciates at a rate of 20% each year. What is the percentage rate of depreciation each year?

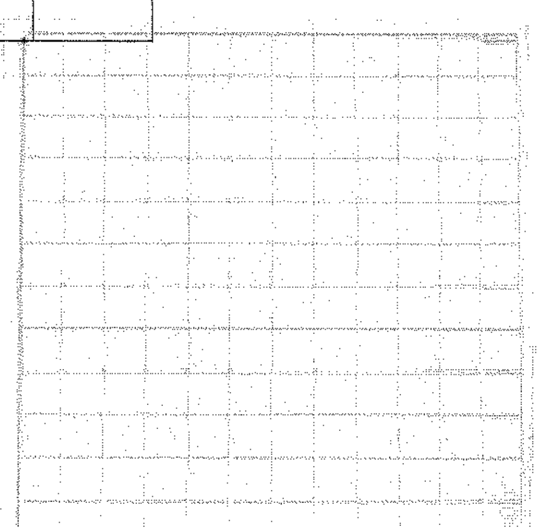
The percentage rate of depreciation is 20%, which means that 80% of the value of the car remains every year. We can calculate this percentage rate by subtracting 20% from 100% in order to calculate the value remaining of 80% each year.

What is the initial value of the car? $____________
What is the percentage rate? 100% - 20% = ___% each year

Let us look at the depreciation data over a 5 year period of time (rounded to the nearest dollar).

Number of Years	0	1	2	3	4	5
Value of the Car	20,000	16,000			8,192	

Graph the table on the graph to the right.



Day 1 Notes

Write an explicit equation for the data in order to calculate the value of deprecation for any year.

$$Y = \text{initial value } (1 - \text{percentage rate of depreciation})^{\text{time}}$$

$$Y = 20{,}000(1 - 0.20)^x$$

$$Y = 20{,}000(0.80)^x$$

Use this equation to find the depreciated value of the car for year 8.

$$Y = 20{,}000(0.80)^8 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

When will the depreciated value of the car be worth $5000? ________________

Estimate this value to the nearest tenth of a year with your calculator by inputting the equation in your calculator and looking at the table.

YOUR TURN

Matt bought a new car at a cost of $25,000. The car depreciates approximately 15% of its value each year.

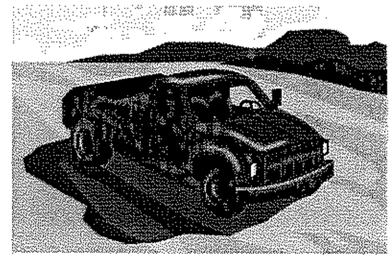
a.) What is the percentage **rate of depreciation** for the value of this car?
(Remember that the percentage rate of depreciation is $0 < b < 1$.)

b.) Write an equation to model the decay value of this car.
$y =$
where y is the value of the car; x is the number of years since new purchase

c.) What will the car be worth in 10 years?

Day 1 Practice

Guided Practice with Depreciation Problems



The cost of a new truck is $32,000. It depreciates at a rate of 15% per year. This means that it loses 15% of each value each year.

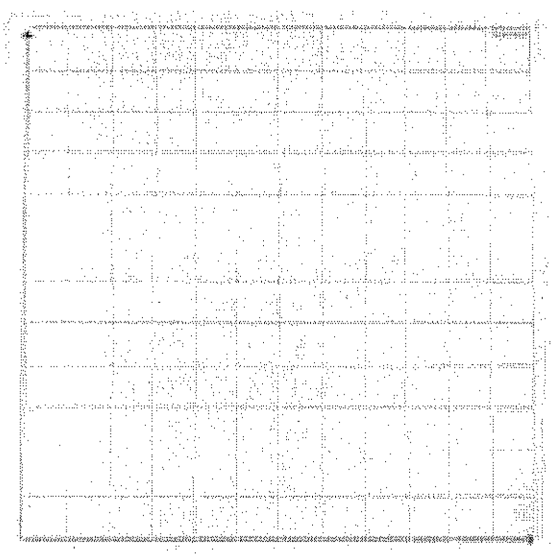
Tasks:

- Draw the graph of the truck's value against time in year.
- Find the formula that gives the value of the truck in terms of time.
- Find the value of the truck when it is four years old.
- Estimate when the truck will be worth half of its value (about $16,000).

Let's start by making a table of values. To fill in the values we start with 32,000 at time $t = 0$. Then we multiply the value of the car by 85% for each passing year. (Since the car looses 15% of its value, which means that it keeps 85% of its value). Remember that 85% means that we multiply by the decimal 0.85.

Number of Years	0	1	2	3	4
Value of the Truck	32,000				

Graph the data on the coordinate grid below. Remember to label your axes.



Now let us write the equation for the data.

Initial value: ____________

Percentage rate of depreciation: ____________

Equation: ______________________________

Use the equation to determine the value of the truck when it is 4 years old.

Value of the 4 year old truck: __________________________

Compare this value with the value in the data table. It should be the same value if your equation is correct.

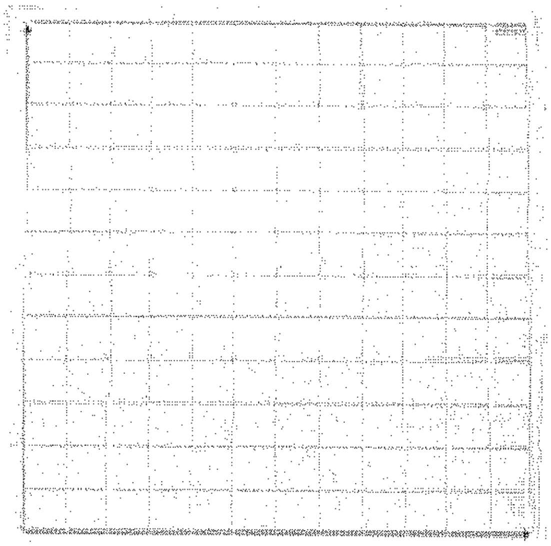
Day 1 Practice

Use the table, graph, equation, or graphing calculator to estimate the time it will take for the truck to worth half of its initial value. ______________________________

Try a few more:

1) The cost of a new ATV (all-terrain vehicle) is $7200. It depreciates at 18% per year. Draw the graph of the vehicle's value against time in years. Find the formula that gives the value of the ATV in terms of time. Find the value of the ATV when it is ten year old.

Number of Years	0	1	2	3	4
Value of the ATV					



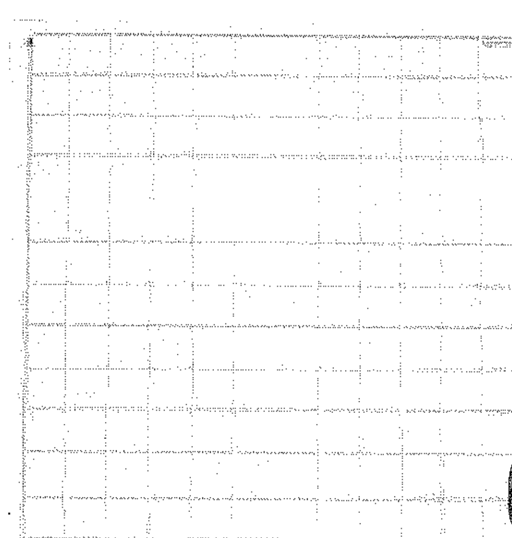
2) A tool & die business purchased a piece of equipment of $250,000. The value of the equipment depreciates at a rate of 12% each year.

   a. Write an exponential decay equation for the value of equipment.

   b. What is the value of equipment after 5 years?

   c. Make a table and graph the model.

Number of Years	0	1	2	3	4
Value of the Equipment					



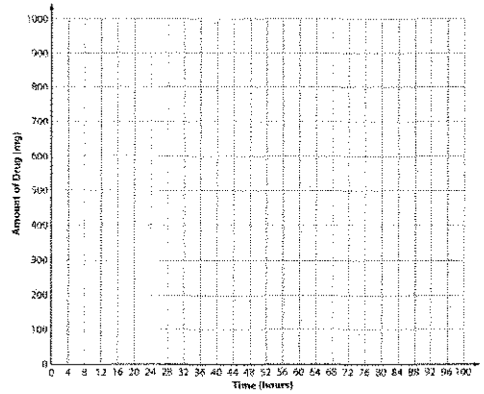
d. Estimate when the equipment will have a value of $70,000.

Other Drug Filtering Problems

1. Assume that your kidneys can filter out 10% of a drug in your blood every 6 hours. You take one 200-milligram dose of the drug. Fill in the table showing the amount of the drug in your blood as a function of time. The first two data points are already completed. Round each value to the nearest milligram.

TIME SINCE TAKING THE DRUG (HR)	AMOUNT OF DRUG IN YOUR BLOOD (MG)
0	200
6	180
12	
18	
24	
30	
36	
42	
48	
54	
60	
66	

Graph the data below.



A. How many milligrams of the drug are in your blood after 2 days?

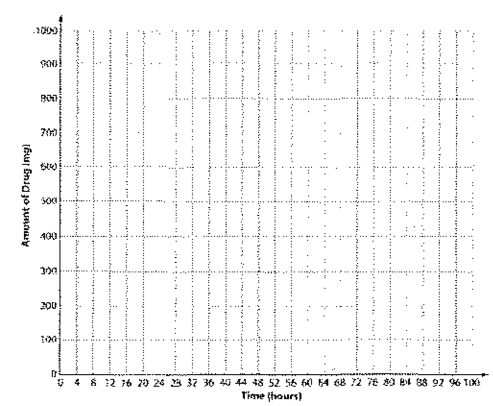
B. A blood test is able to detect the presence of the drug if there is at least 0.1 mg in your blood. How many days will it take before the test will come back negative? Explain your answer.

Day 1 Homework

2. Have students calculate the amount of drug remaining in the blood in the original lesson, but instead of taking just one dose of the drug, now take a new dose of 1000 mg every four hours. Assume the kidneys can still filter out 25% of the drug in your blood every four hours. Have students make a complete a table and graph of this situation. How does the results differ from the situation explored during the main lesson? Have students use their data table and graph to justify their response.

TIME SINCE TAKING THE DRUG (HR)	AMOUNT OF DRUG IN YOUR BLOOD (MG)
0	1000
4	1750
8	
12	
16	
20	
24	
28	
32	
36	
40	
44	
48	

Graph the data below.

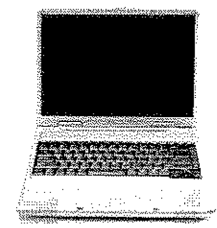


A. How many milligrams of the drug are in your blood after 2 days?

B. A blood test is able to detect the presence of the drug if there is at least 0.1 mg in your blood. How many days will it take before the test will come back negative? Explain your answer.

Day 1 Homework

Independent Practice with Depreciation Problems



Use equations, graph, or tables to find the solutions to the problems below.

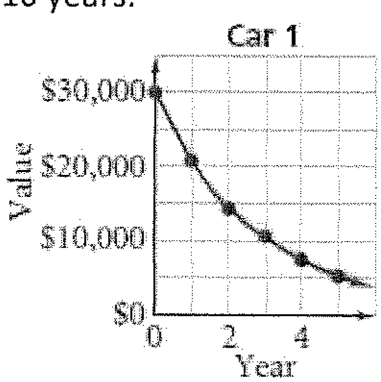
1) A computer valued at $6500 depreciates at the rate of 14.3% per year. Write a function that models the value of the computer. Find the value of the computer after three years.

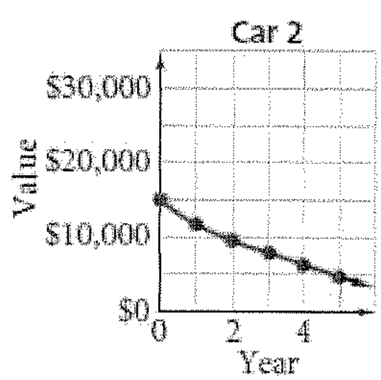
2) A new truck that sells for $29,000 depreciates 12% each year. Write a function that models the value of the truck. Find the value of the truck after 7 years.

3) A new car that sells for $18,000 depreciates 25% each year. Write a function that models the value of the car. Find the value of the car after 4 years.

4) You purchased a car for $19,500. The car will depreciate at a rate of 12% each year. Write a formula to represent the value of the car after x number of years. Find the value of the car after 4 years.

5) Each graph below shows the expected decrease in a car's value over the next five years. Write a function to model each car's depreciation. Determine which car will be worth more after 10 years.





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Day 2 Notes/Practice

More Half-Life Problems

Most things are composed of stable atoms. However, the atoms in radioactive substances are unstable and the break down in a process called radioactive decay. The rate of decay varies from substance to substance. The term **half-life** refers to the time it takes for half of the atoms in a radioactive substance to decay. For example, the half-life of carbon-11 is 20 minutes. This means that 2,000 carbon-11 atoms will be reduced to 1,000 carbon-11 atoms in 20 minutes, and to 500 carbon-11 atoms in 40 minutes.

Half-lives vary from a fraction of a second to billions of years. For example, the half-life of polonium-214 is 0.00016 seconds. The half-life of rubidium-87 is 49 billion years.

In the problems below, write an exponential decay function in order to find the solution to each problem. (Use function notation)

1) Hg-197 is used in kidney scans and it has a half-life of 64.128 hours. Write the exponential decay function for a 12-mg sample. Find the amount remaining after 72 hours.

2) Sr-85 is used in bone scans and is has a half-life of 64.9 days. Write the exponential decay function for an 8-mg sample. Find the amount remaining after 100 days.

3) I-123 is used in thyroid scans and has a half-life of 13.2 hours. Write the exponential decay function for an 45-mg sample. Find the amount remaining after 5 hours.

4) A decaying radioactive ore originally weighs 27 grams and is reduced to 18 grams in 1,000 years. How much will be left in 3,000 years? Write an exponential decay function in order to find the solution.

5) Some radioactive ore which weighed 20 grams 200 years ago has been reduced to 12 grams today. How much will be left 400 years from now? Write an exponential decay function in order to find the solution.

Day 2 Homework

Independent Practice Half-life Problems

Recall: The half-life of a radioactive substance is the time it takes for half of the material to decay. You are encouraged to make a table in order to generate some of the data for each problem situation below. Solve the following half-life problems by writing an equation and using the equation to find the solution. Make sure you find the initial value for each equation. The first problem has been **partially worked** in order to help you with the remaining problems.

1) A hospital prepared a 100-mg supply of technetium-99m, which has a half-life of 6 hours. Use the table below to help you understand how much of technetium-99m is left at the end of a 6-hour interval for 36 hours. Use this to help write an exponential function to find the amount of technetium-99m that remains after 75 hours.

The amount of technetium-99m is reduced by one half each 6 hours as shown in the table below. Fill in the missing information in the table and in the equation below.

Number of 6-hour Intervals	0	1	2	3	4	5	6
Number of Hours Elapsed	0	6		18	24		36
Amount of Technetium-99m (mg)	100	50	25			3.13	

The amount of technetium-99m is an exponential function of the number of half-lives. The initial amount is _____ mg. The decay factor is _____. One half-life equals 6 hours.

Y = ____

X = the number of hours elapsed. So, (1/6)x = the number of half-lives.

Equation: y = ______(1/2)$^{—}$

Use your equation to find the solution to the question.

Y = _____$(1/2)^{(1/6)x}$

Y = 100 $(1/2)^{(1/6)(75)}$

Y = _______________

After 75 hours, about ___________ mg of technetium-99 remains.

Day 2 Homework

Use a similar format in order to find the equations and solutions of the 4 remaining problems.

2) Arsenic-74 is used to locate brain tumors. It has a half-life of 17.5 days. Write an exponential decay function for a 90-day sample. Use the function to find the amount remaining after 6 days. (Hint: Make a table to help you understand the data.)

3) Phospohoru-32 is used to study a plant's use of fertilizer. It has a half-life of 14.3 days. Write the exponential decay function of a 50-mg sample. Find the amount of phosphorus-32 remaining after 84 days.

4) Iodine-131 is used to find leaks in water pipes. It has a half-life of 8.14 days. Write the exponential decay function for a 200-mg sample. Find the amount of iodine-131 remaining after 72 days.

5) Carbon-14 is used to determine the age of artifacts in carbon dating. It has a half-life of 5730 years. Write the exponential decay function for a 24-mg sample. Find the amount of carbon-14 remaining after 30 millennia (1 millennium – 1000 years).

Compound Interest Guided Notes

$$P = P_0\left(1 + \frac{r}{n}\right)^{nt}$$

For graduation I received a total of $2500. I want to put it into a CD with an interest rate of 4.6% compounded monthly. Using this equation, write a function that models the balance of the CD.

P_0—original /principle amount

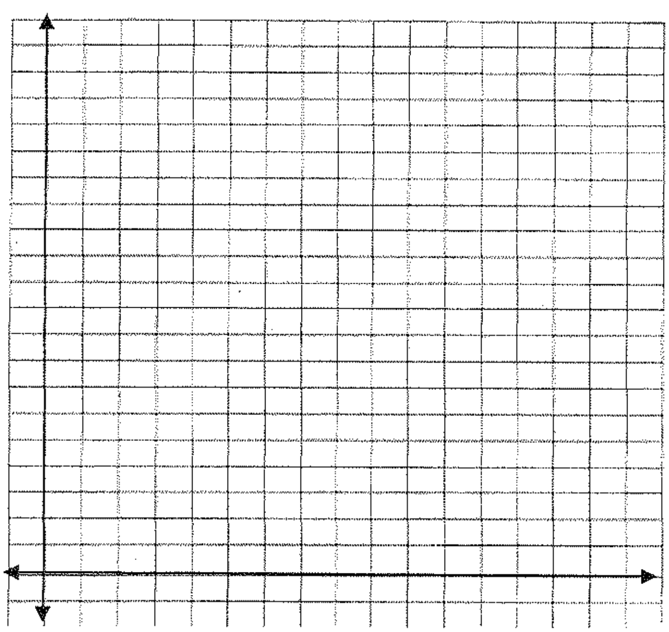
r—the interest rate

n—the number of times the interest is compounded yearly

t—the amount of time

P—the current balance

____



x	y

What is the balance after 5 years?

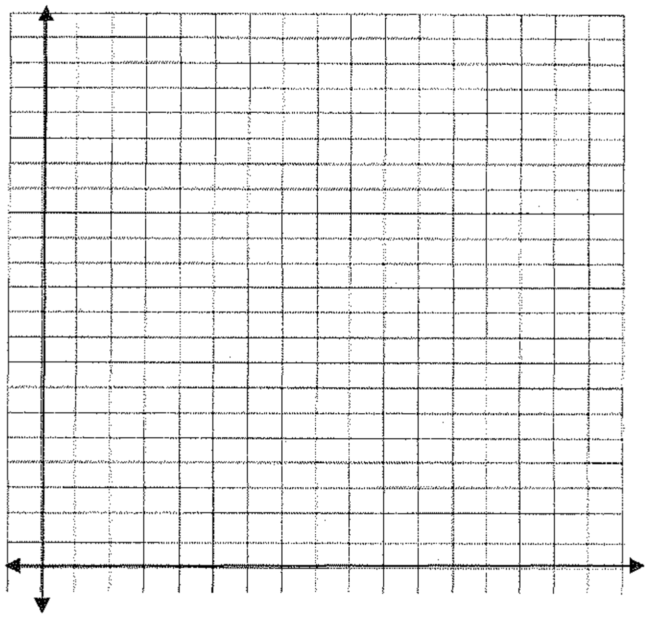
____

What is the formula for this?

____

12

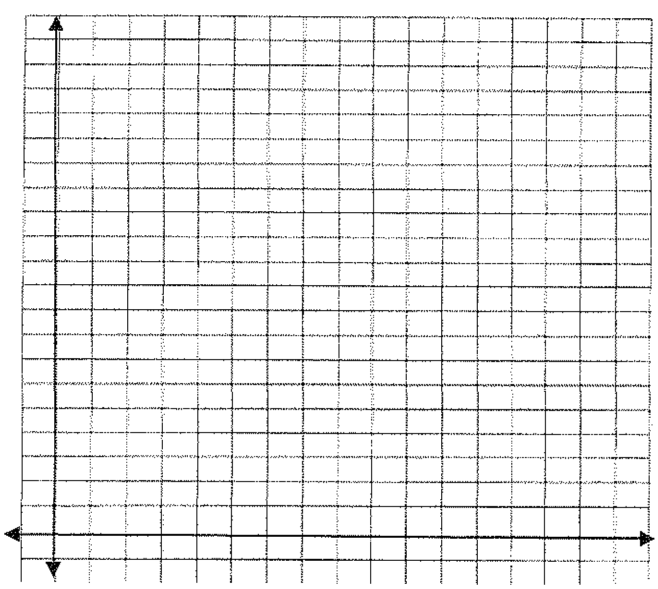
The principal amount of deposit is $1640. It has an interest rate of 3.2% compounded quarterly. Write a function and graph it. ______________________



From the graph, give an approximation of the balance after 3 years. ______________

What is the balance to the nearest hundredth after 6 years? ______________

The principle amount of deposit is $1350. It has an interest rate of 4.6% compounded monthly. Write a function and graph it. ______________________



From the graph, what is the approximate balance after 7 years?

____

What is the balance to the nearest hundredth after 10 years?

____

Day 4 Notes

More Compound Interest . . . Making even more money by investing

The scenario is that you have $5000 to invest and you want to know which of the following investment situations will give you the most money at the end of 5 years. The interest rate for all of the situations is 6%.

1. Calculate the investment if it is compounded annually.

2. Calculate the investment if it is compounded semi-annually (twice a year).

3. Calculate the investment if it is compounded quarterly (four times a year).

4. Calculate the investment if it is compounded monthly (12 times a year).

5. Calculate the investment if it is compounded daily (365 times a year).

6. What did you discover? Which situation will give you the most? Which situation is the most realistic for banks? Which situation is the most realistic for credit card companies? Explain your reasoning.

Day 4 Homework

Independent Practice with Compound Interest

Write an explicit equation for each problem situation in order to find the solution. Find the solution using your equation.

1) An investment of $75,000 increases at a rate of 12.5% per year. Find the value of the investment after 30 years. How much more would you have if the interest is compounded quarterly?

2) Suppose you invest $5000 at an annual interest of 7%, compounded semi-annually. How much will you have in the account after 10 years? Determine how much more you would have if the interest were compounded monthly.

3) Lisa invested $1000 into an account that pays 4% interest compounded monthly. If this account is for her newborn, how much will the account be worth on his 21st birthday, which is exactly 21 years from now?

4) Mr. Jackson wants to open up a savings account. He has looked at two different banks. Bank 1 is offering a rate of 5.5% compounded quarterly. Bank 2 is offering an account that has a rate of 8%, but is only compounded semi-annually. Mr. Jackson puts $6,000 in an account and wants to take it out for his retirement in 10 years. Which bank will give him the most money back?

5) Mason deposited $2,000 into a savings account that pay an annual interest rate of 9% compounded annually. Determine the amount of money in the savings account after 1 year, 5 years, 10 years and 20 years. Using the calculated values, construct a graph.

Day 5 Notes

Population Growth Problems

One practical application of exponential growth is predicting growth of various populations. You have already looked at problems involving population growth when looking at the Alien and Monster lesson, and other problems encountered in lessons or homework. Some of the growth factors were whole numbers and some were fractional. In this lesson we will explore more fractional growth factors.

Aussie Rabbits

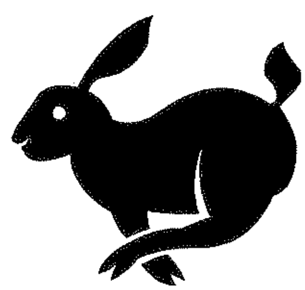
In 1859, a small number of rabbits were introduced to Australia by English Settlers. The rabbits had no natural predators in Australia, so they reproduced rapidly and quickly became a serious problem for sheep and cattle, eating the grasses intended for them.

In the mid-1900s, there were more than 300 million rabbits in Australia. The damage they caused cost Australian agriculture $600 million per year. There have been many attempts to curb Australia's rabbit population. In 1995, a deadly rabbit disease was deliberately spread, reducing the rabbit population by about half. However, because rabbits are developing immunity to the disease, the effects of this measure may not last.

If biologists had counted the rabbits Australia in the years after they were introduced, they might have collected data like these:

Growth of Rabbit Population

Time (Year)	Population
0	100
1	180
2	325
3	583
4	1,050



A. The table shows the rabbit population growing exponentially.
   1. What is the growth factor? Explain how you found your answer.
   2. Assume this growth pattern continued. Write an equation for the rabbit population p for any year n after the rabbits are first counted. Explain what the numbers in your equation represent.

Day 5 Notes

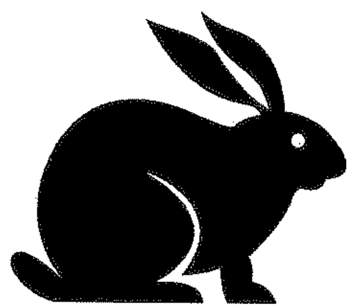
3. How many rabbits will there be after 10 years? How many will there be after 25 years? After 50 years?
4. In how many years will the rabbit population exceed one million?

B. Suppose that, during a different time period, the rabbit population could be predicted by the equation $p = 15(1.2)^n$, where p is the population in millions, and n is the number of years.
1. What is the growth factor?
2. What was the initial population?
3. In how many years will the population double from the initial population?
4. What will the population be after 3 years? After how many more years will the population at 3 years double?
5. What will the population be after 10 years? After how many more years will the population at 10 years double?
6. How do the doubling time for parts (3) – (5) compare? Do you think the doubling time will be the same for this relationship no matter where you start to count?

The yearly growth factor for the table above is about 1.8. Suppose the population data fit the equation $p = 100(1.8)^n$ exactly. Then its table would look like the one below.

Rabbit Population Growth

Time (Year)	Population
0	100
1	100 • 1.8 = 180
2	180 • 1.8 = 324
3	324 • 1.8 = 583.2
4	583.2 • 1.8 = 1,049.76



The growth factor of 1.8 is the number by which the population for year n is multiplied to get the population for the next year, $n + 1$. You can think of the growth factor in terms of a percent change. To find the percent change, compare the difference in population for two consecutive years, n and $n + 1$, with the population of year n.

Here's the way to think about this percent change.

Day 5 Notes

- From year 0 to year 1, the percent change is (180 – 100)/100 = 80/100 = 80%. The population of 100 rabbits in year 0 *increased by* 80%, resulting in 100 rabbits • 80% = 80 additional rabbits.
- From year 1 to year 2, the percent change is (324 – 180)/180 = 144/180 = 80%. The population of 180 rabbits in year 1 *increased by* 80%, resulting in 180 rabbits • 80% = 144 additional rabbits.

So, ask yourself the following questions:

- If we are increasing the population of rabbits 80% each year, where does the 1 come from in the 1.8 growth rate in the explicit equation: $p = 100(1.8)^n$?
- How can the equation $p = 100(1.8)^n$ be written as a NOW-NEXT equation?

YOUR TURN

Let's look at another population . . . the wolves of northern Michigan.

In parts of the United States, wolves are being reintroduced to wilderness areas where they had become extinct. Suppose 20 wolves are released in northern Michigan, and the yearly growth factor for this population is expected to be 1.2.

1. Make a table showing the projected number of wolves at the end of each of the first six years.

2. Write an explicit equation in function form that models the growth of the wolf population.

3. Using either equation, how long will it take for the new wolf population to exceed 100?

4. Using either equation, how many wolves will there be in 10 years? 15 years? 25 years?

Day 5 Practice

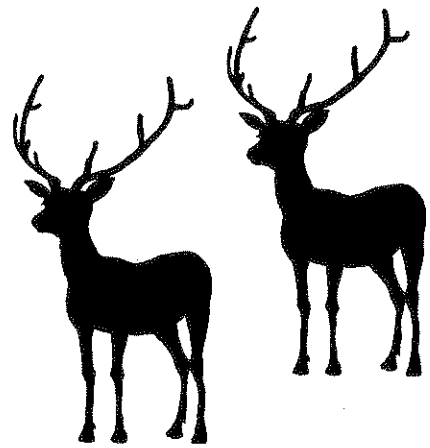
Population Growth and Other Word Problems

The Elk Population

1) The table show that the elk population in a state forest is growing exponentially. What is the growth factor? Explain.

Growth of Elk Population

Time (Year)	Population
0	30
1	57
2	108
3	206
4	391
5	743



2) Suppose this growth pattern continues. How many elk will these be after 10 years? How many elk will there be after 15 years?

3) Write a NOW-NEXT equation you could use to predict the elk population p for any year n after the elk were first counted.

4) Use this equation to write an explicit equation in function notation to predict the elk population p for any year n after the elk were first counted.

5) In how many years will the elk population exceed one million?

For problems 6 and 7, write an explicit equation in function notation before find the solution(s) to the problems.

6) Suppose there are 100 trout in a lake and the yearly growth factor for the population is 1.5. How long will it take for the number of trout to double?

Day 5 Practice

7) Suppose there are 500,000 squirrels in a forest and the growth factor for the population is 1.6 per year. Write an equation you could use to find the squirrel population p in n years.

8) Currently, 1,000 students attend East Garner IB Magnet Middle School. The school can accommodate 1,300 students. The school board estimates that the student population will grow by 5% per year for the next several years.
   a) In how many years will the population outgrow the present building?

   b) Suppose the school limits its growth to 50 students per year. How many years will it take for the population to outgrow the school?

9) Suppose that, for several years, the number of radios sold in the U.S. increased by 3% each year.
   a) Suppose one million radios sold in the first year of this time period. About how many radios sold in each of the next 6 years?

   b) Suppose only 100,000 radios sold in the first year. About how many radios sold in each of the next 6 years?

10) Suppose a movie ticket costs about $7, and inflation causes ticket prices to increase by 4.5% a year for the next several years.
   a) At this rate, how much will tickets cost 5 years from now?

   b) How much will a ticket cost 10 years from now?

   c) How much will a ticket cost 30 years from now?

   d) When will a ticket cost $25?

Day 5 Homework

Independent Practice with Population Growth and Other Word Problems

Write an explicit equation in function notation to assist in solving the following problems.

1) Omar made the following calculation to predict the value of his baseball card collection several years from now:

$$\text{Value} = \$130 \bullet 1.07 \bullet 1.07 \bullet 1.07 \bullet 1.07 \bullet 1.07$$

   a) What initial value, growth rate, growth factor, and number of years is Omar assuming?

   b) Write the equation modeling this problem in function notation.

   c) If the value continues to increase at this rate, how much would the collection be worth in three more years?

2) Carlos, Latanya, and Mila work in a biology laboratory. Each of them is responsible for a population of mice. The growth factor for Carlos's population of mice is 8/7. The growth factor for Latanya's population of mice is 3. The growth factor for Mila's population of mice is 125%.

   a) How mice are reproducing fastest?

   b) Whose mice are reproducing slowest?

3) A worker currently receives a yearly salary of $20,000.

   a) Find the dollar values of a 3%, 4%, and 6% raise for this worker.

   b) Find the worker's new annual salary for each raise in part a.

   c) You can find the new salary after a 3% raise in two ways:

      $20,000 + 3% of $20,000 OR 103% of $20,000

      Explain why these two methods give the same result.

Day 5 Homework

4) Kwan cuts lawns every summer to make money. One of her customers offers to give her a 3% raise next summer and a 4% raise the summer after that. Kwan says she would prefer to get a 4% raise next summer and a 3% raise the summer after that. She claims she will earn more money this way. Is she correct? Explain why or why not.

5) In 1990, the population of the U.S. was about 250 million and was growing exponentially at a rate of about 1% per year.
   a) At this growth rate, what will the population of the U.S. be in the year 2010?

   b) At this rate, how long will it take the population to double?

   c) Do you think the predictions in parts a and b are accurate? Explain.

   d) The population in 2000 was about 282 million. How accurate was the growth rate?

6) The Greens bought a condominium for $83,000. Assuming that its value will appreciate 6% per year, how much will the condo be worth in 5 years when the Greens are ready to move?

7) Ten years ago, Mr. and Mrs. Boyce bought a house for $96,000. Their home is now worth $125,000. Assuming a steady rate of growth, what was the annual rate of appreciation?

8) In 2000, the number of people worldwide living with HIV/AIDS was estimated at more than 36 million. That number was growing at an annual rate of about 15%.

   a) Make a table showing the projected number of people around the world living with HIV/AIDS in each of the ten years after 2000, assuming the growth rate remains 15% per year.

b) Write two different kinds of rules that could be used to estimate the number of people living with HIV/AIDS at any time in the future.

c) Use the rules from part b to estimate the number of people living with HIV/AIDS in 2015.

d) What factors might make the estimate of part c an inaccurate forecast?

9) Hypothermia is a life-threatening condition in which body temperatures fall well below the norm of 98.6°F. However, because chilling causes normal body functions to slow down, doctors are exploring ways to use hypothermia as a technique for extending time of delicate operations like brain surgery.

a) The following table gives experimental data illustrating the relationship between body temperature and brain activity.

Body Temperature (in °F)	50	59	68	77	86	98.6
Brain Activity (% Normal)	11	16	24	37	52	100

b) Plot the table data and find an explicit equation that models the pattern in these data relating brain activity level to body temperature.

c) Use your rules to estimate the level of brain activity at a body temperature of 39°F, the lowest temperature used in surgery experiments on pigs, dog, and baboons.

d) Find the range of body temperatures at which brain activity is predicted to be about 75% of normal levels.

Word Problem Review Worksheet

Use your knowledge of exponential functions to answer the following questions. Show your work, especially equations written to assist in finding solutions.

1. Which is the best investment if the money in each case is invested for three years?
    a. $5,000 at 8% compounded monthly
    b. $5,000 at 8.2% compounded annually
    c. $5,000 at 8.1% compounded semiannually

2. The population of a bacteria culture doubles after 1.5 hours. An experiment begins with 620 bacteria. Determine the number of bacteria after

a. 3 hours	
b. 10 hours	
c. 10 hours	
d. 10 hours	
e. 10 hours	
f. 10 hours	

3. The half-life of a radioactive material is about 2 years. How much of a 5-kg sample of this material would remain after: (This is not multiple choice, answer all for scenarios)
    a. 4 years b. 3 years c. 5.5 years d. 18 months

4. The population of Littleton is currently 23,000. Assume that Littleton's exponential growth rate is 2% per year.
   a. Copy and complete the table by predicting the population for the next six years.

Time (years)	0	1	2	3	4	5	6
Population	23,000						

   b. Graph the data.
   c. Create the equation to model the equation.
   d. Use your equation to predict the population in 10 years.
   e. Use your graph to estimate how long it will take the population will reach 30,000.
   f. Predict the population of Littleton after 10 years if the growth rate is 3%.

5. A population, P, is increasing exponentially. At time $t = 0$, the population is 35,000. In 10 years, the population is 44,400.
   a. Find a in $P = a(b)^t$.
   b. Use the value of b that you calculated, write an equation that models the population, P, after t years.
   c. Using your equation, find when the population reaches 100,000.

6. A bacteria culture starts with 3,000 bacteria and grows to a population of 12,000 after 3 hours.
   a. Find the doubling period.
   b. Find the population after t hours.
   c. Determine the number of bacteria after 8 hours.
   d. Determine the number of bacteria after 1 hour.

7. The half-life of caffeine in a child's system when a child eats or drinks something with caffeine in it is 2.5 hour. How much caffeine would remain in a child's body if the child ate a chocolate bar with 20 mg of caffeine 8 hours before?

8. Twelve grams of tritium decays to 9.25 grams in 2.5 years. Use a method to estimate the half-life of tritium.

9. A radioactive form of uranium has a half-life of 2.5×10^5 years.
   a. Find the remaining mass of 1 gram sample after t years.

   b. Determine the remaining mass of this sample after 5000 years.

10. The half-life of carbon-14 is about 5370 years. What percent of the original carbon-14 would you expect to find in a sample after 2500 years?

11. An old stamp is currently worth $60. The stamp's value will grow exponentially 15% per year.
    a. What will the value of the stamp be in 8 years?

    b. When will the value of the stamp be worth 3 times the initial value?

12. A photocopier, which originally costs $500,000, depreciates exponentially by 10% each year.
    a. What will the photocopier's value be worth in 5 years?

    b. When will the photocopier's value be $175,000?

13. After an accident at a nuclear power plant, which caused a radiation leak, the radiation level at the accident was 950 roentgens. Five hours later, the radiation level was 800 roentgens. Radiation levels decay exponentially. Find the rate of decay.

14. Annie bought a new car for $35,000 and sold it 5 years later for $18, 475. Assume that the value of the vehicle depreciates exponentially. Calculate the rate of depreciation per year.

15. Mark invests $500 in a savings plan that pays interest, which is compounded monthly. At the end of 10 years, his initial investment is worth $909.70. What interest rate did the plan pay?

16. An exponential function is expressed in the form $y = a(b)^x$. How can you tell whether the relation represents growth or decay?

17. The population of a small town increases exponentially. In 1999, the population was 16,000 and in 2002 it was 60,000. What will the population be in 2010?

18. In 1996, Ontario's population was about 10.7 million. Ontario's population will be about 13.7 million in 2016.
   a. Calculate the annual growth rate of Ontario's population.
   b. What would Ontario's population have been in 1980?
   c. What have you assumed for part a and part b?

19. During an archaeological dig, Selma found a tool that resembled a small hatchet with a wooded handle.
   a. Carbon-14 has a half-life of about 5370 years. Explain what this means.
   b. Create an equation that relates the percent of carbon-14 remaining to the tool's age. Explain what each part of the equation represents.
   c. Explain how you can tell from the equation that the amount of carbon-14 is decreasing.
   d. Explain how you can tell from the equation that the amount of carbon-14 is decreasing exponentially.