CCM1B - Unit 1: Exponential Functions

Name: ______________________

Objectives:

Essential Questions

By the end of this unit, students will be able to answer the following questions:

1. What considerations should be taken into account when determining the boundaries and scales of a graph?
2. What are the key features of an exponential function?
3. When given one of the four forms of information, what should be taken into consideration when determining the best function to model the situation? Explain.
4. Why is a multiplicative rate of change the key feature of an exponential function and how is it revealed in the different forms of this function (verbal, graph, table, equation)?
5. How do you determine the best model for a data pattern?
6. When given a sequence, how do I identify whether it is arithmetic or geometric and how do I write a rule for the sequence?

Enduring Understandings

By the end of the unit, the student will understand that:

1. The context of a problem determines the domain and range as well as an appropriate scale of an exponential function.
2. The equation of an exponential function reveals the initial value and the rate of change for a given situation.
3. The points on the graph of the equation represent the solutions to exponential relationships.
4. The exponential relationship between two quantities can be expressed graphically and with a symbolic rule.
5. Exponential functions model real world problems, of growth and decay, such as monetary growth, population increases or decreases, car values, half-life, etc.
6. Linear and exponential functions exhibit different characteristics.
7. One type of function does not fit all situations in life.

Exponential functions can be written as explicit expressions or using the recursive process.

Date	Day	Lesson	Assignment
Wed. 1/23	1	Policies and Procedures	
Thurs. 1/24	2	Exponent Rules	
Fri. 1/25	3	Exponential Growth & Decay	
Mon. 1/28	4	Translating & Graphing Exponential Functions	
Tues 1/29	5	Sequences (Arithmetic vs. Geometric)	
Wed. 1/30	6	Sequences to Tables & Graphs	
Thurs. 1/31	7	**Unit 1 Quiz:** **Exponent Rules, Graphing Exponential Functions, and Sequences**	
Fri. 2/1	8	One Grain of Rice Activity (Sequences to Recursive Formulas)	

Mon. 2/11	9	Bacteria Growth Activity (Recursive to Explicit)	
Tues. 2/12	10	Review of Unit 1	
Wed. 2/13	11	**Unit 1 Test**	

Homework Grade:

Properties of Exponents - Day 2 Notes

Discuss with a partner: What is the *meaning* of the expression 2^3? What is the *value* of the expression 2^3? How do you know?

Recall that in an exponential expression of the form b^n, b is called the base and n is called the exponent. You can say that b is raised to the n^{th} power. For example, 5^4 would be read as "five raised to the fourth power." But what does this *mean*?

In expanded form, we would write $5^4 = 5 \cdot 5 \cdot 5 \cdot 5$. So the exponent is short-hand notation for repeated multiplication. Raising five to the fourth power means multiplying 5 times itself four times.

So what about $5^{\frac{1}{2}}$? How can we multiply 5 by itself one-half times?

In today's investigation, we will find the meaning behind this expression.

But first let's review what else you know about exponents. In previous courses, you learned properties for rewriting expressions involving exponents. Let's look at some patterns and see if we can rediscover those properties.

Multiplying exponential expressions with like bases:
Work with a partner to fill in the blanks, and then study the pattern to determine the property, or shortcut, for multiplying exponential expressions with like bases.

$$2^3 \cdot 2^4 = (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7$$
$$x^5 \cdot x^{12} = (x \cdot x \cdot x \cdot x \cdot x)(\qquad\qquad\qquad\qquad) =$$
$$10^2 \cdot 10^5 = (\qquad\qquad)(\qquad\qquad\qquad) =$$

$$x^m \cdot x^n =$$

When multiplying exponential expressions with like bases, you should ______________ the exponents.

Dividing exponential expressions with like bases
Work with a partner to fill in the blanks, and then study the pattern to determine the property, or shortcut, for dividing exponential expressions with like bases.

$$\frac{4^5}{4^2} = \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4} = \frac{4}{4} \cdot \frac{4}{4} \cdot 4 \cdot 4 \cdot 4 = 1 \cdot 1 \cdot 4 \cdot 4 \cdot 4 = 4 \cdot 4 \cdot 4 = 4^3$$

$$\frac{x^8}{x^6} = \frac{\qquad\qquad\qquad\qquad}{\qquad} =$$

$$\frac{(0.94)^{15}}{(0.94)^5} = \frac{\qquad\qquad\qquad\qquad}{\qquad} =$$

$$\frac{x^m}{x^n} =$$

When dividing exponential expressions with like bases, you should ________________ the exponents.

Raising a power to a power

Work with a partner to fill in the blanks, and then study the pattern to determine the property, or shortcut, for raising an exponential expression to a power.

$$(2^3)^4 = (2^3)(2^3)(2^3)(2^3) = 2^{3+3+3+3} = 2^{12}$$

$$(x^5)^2 = (x^5)(\quad) = x^{\_\_+\_\_} = x$$

$$(10^2)^6 = (\quad)(\quad)(\quad)(\quad)(\quad)(\quad) =$$

$$(x^m)^n =$$

When raising an exponential expression to a power, you should __________________ the exponents.

Negative exponents

Work with a partner to fill in the blanks, and then study the pattern to determine the meaning of a negative exponent.

$$\frac{2^3}{2^4} = \frac{2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{2}{2} \cdot \frac{2}{2} \cdot \frac{2}{2} \cdot \frac{1}{2} = 1 \cdot 1 \cdot 1 \cdot \frac{1}{2} = \frac{1}{2^1}$$

$$\frac{4^5}{4^7} = \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} = \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ = \_$$

$$\frac{x^3}{x^8} = \_\_\_\_\_\_\_\_ = \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ \cdot \_ = \_$$

If we apply the previous rule about dividing exponential expressions with like bases, each of the above problems would look like:

$$2^{3-4} = 2^{-1}$$

$$4^{5-7} = 4$$

$$x^{3-8} = x$$

Find the following values for powers of 3 and record in the table below. Use a calculator if needed. Convert any decimal values to fraction form.

Power of 3	Value
3^2	
3^1	
3^0	
3^{-1}	
3^{-2}	

What is the meaning of a negative exponent?

$$x^{-m} =$$

Day 2 Classwork

Check Your Understanding
Fill in the missing parts

Exponents	Written Expression	Expanded Form	Simplified
2^7			
	5 to the 4th power		
		(ab)(ab)(ab)	
			121
$(a^3b^4)^5$			
			$3^8x^8y^8$
$\frac{8x^3y^5}{4xy^2}$			
$(5)^{-2}$			

AAT - Section 2.2

Day 2 Homework
evens only

Name______________________

Exponent Properties - WORKSHEET 2.2 - #1

Date______________ Period____

Simplify.

1) $2^3 \cdot 2^2 \cdot 2^3$

2) $(-3)^3 \cdot (-3)^2$

3) $2^3 \cdot 3^4$

4) $9 \cdot 9^8 \cdot 9^{10}$

5) $3x^3 \cdot 2x^2$

6) $2v^2 \cdot 2v^3 \cdot 3v^2$

7) $2n^3 \cdot n^2 \cdot -3n^3$

8) $nm^3 \cdot m^3n^3$

9) $-3x^2y^3 \cdot -2x^3y^3$

10) $3x^3y^3 \cdot 2x^2y^2 \cdot -2yx^3$

11) $3ba^3c^4 \cdot a^4b^2c^4$

12) $3xzy^3 \cdot 3x^3y^4z^2 \cdot 3yx^3z^4$

13) 3^{-2}

14) $\frac{1}{2^{-4}}$

15) $2nm^{-2}$

16) $4u^{-3}v^{-4}$

17) $4y^{-2}z^4$

18) $h^3j^{-1}k^{-3}$

19) $\frac{5^6}{5^2}$

20) $\frac{5^4}{5^6}$

21) $\frac{p^5}{p^2}$

22) $\frac{3a^4}{a^6}$

23) $-\frac{2x^4y^3}{x^2y^2}$

24) $\frac{4x^2y^6}{3x^4y^3}$

25) $\frac{yx^4z^2}{xy^4z^3}$

26) $-\frac{2x^2y^2}{x^2y^4z^4}$

27) $(2^3)^4$

28) $((-4)^2)^4$

29) $(n^4)^3$

30) $(2x^3)^4$

31) $(m^4n^2)^4$

32) $(-4vu^2)^3$

33) $(4x^2y^2z^4)^2$

34) $(x^4y^2z^4)^4$

4

Day 3 Notes

Exponential Growth and Decay Name Date

Fold your paper in half, and complete the following table. Continue folding your paper in half and recording values in the table until you cannot fold the paper in half anymore.

Based on your results, predict values for the table after 10 folds.

Generalize your results by completing the table for "n" folds.

Number of Folds	Number of Sections	Fraction Each Section's Area is of the Whole Paper
0	1	1
1		
2		
3		
4		
5		
10 (predict)		
n (write a formula)		

① What happens to the # sections as you fold the paper each time?

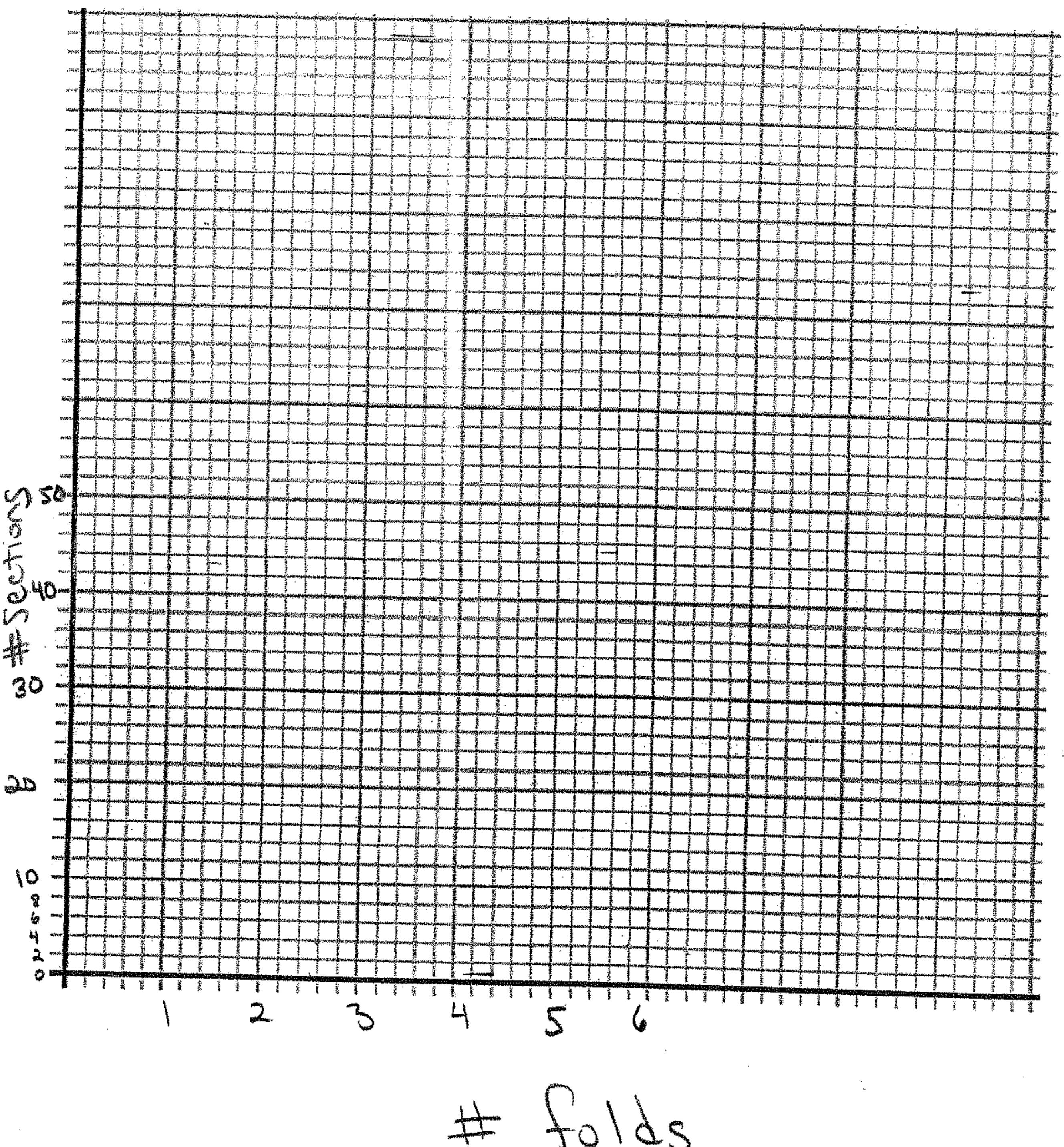
② Assuming you could always keep folding the paper, is there a limit to the # sections that would be formed?

③ What happens to the fraction of the area as you fold the paper each time?

Assuming you could always keep folding the paper, does the fraction of the area ever reach zero?

(5) Plot the data points on the graphs below. Don't forget to label your axes and write your scale.

Number of Sections



This graph represents exponential ____________ . Complete these characteristics

Domain: ____________

Range: ____________

As x increases, y ____________

As x decreases, y ____________

Asymptote at ____________

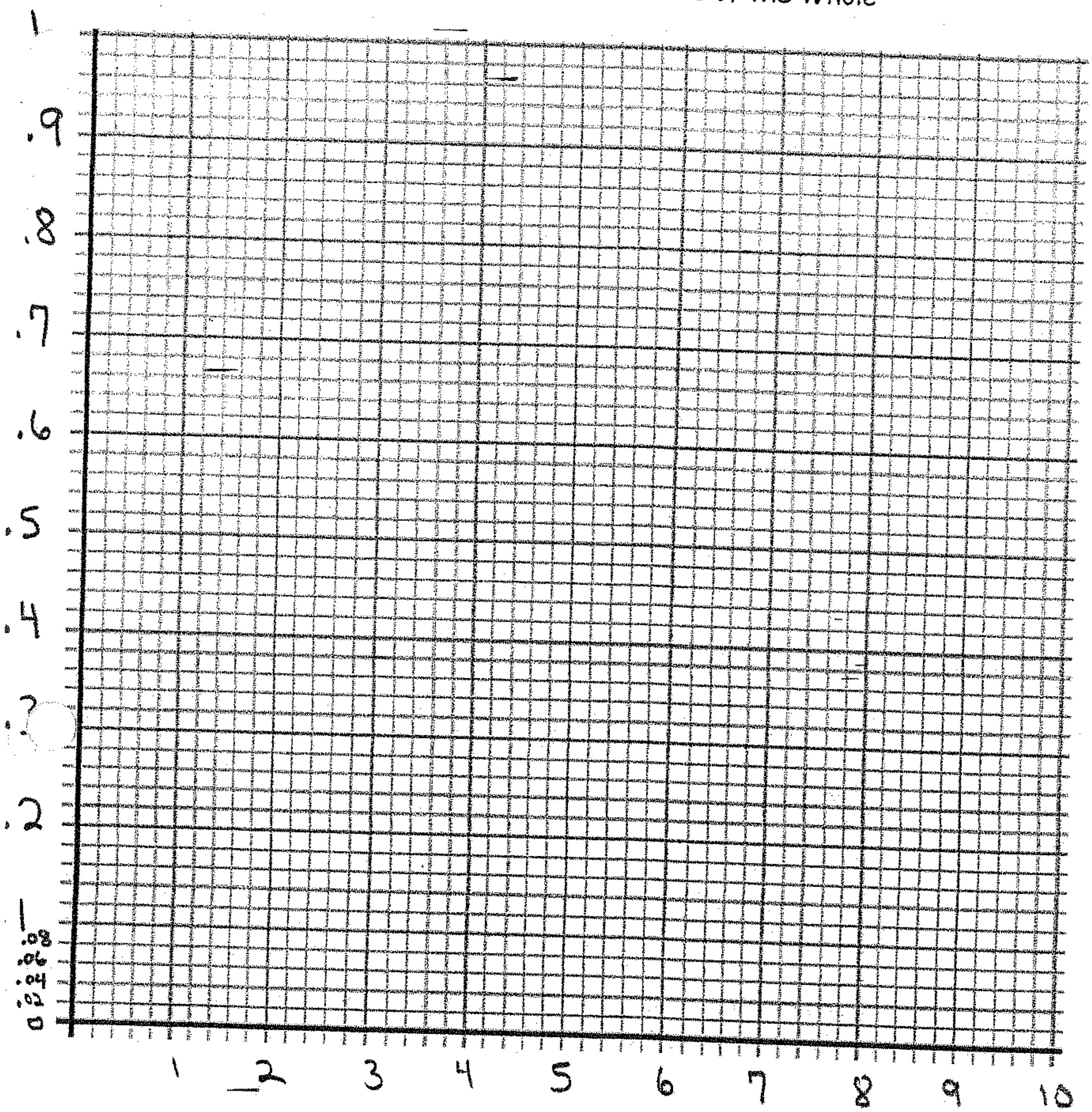
Increasing: ____________

Decreasing: ____________

Rate of Change: ____________

6

Fraction each Section is of the Whole



folds

This graph represents exponential ________. Complete these characteristics.

Domain: ____________________

Range: ____________________

As x increases, y ____________

As x decreases, y ____________

Asymptote at ____________

Increasing: ____________

Decreasing: ____________

Rate of Change: ____________

7

⑥ Use your graphing calculator to draw a rough sketch of the following. State if the function represents exponential growth or decay.

a) $y = 1.3^x$

b) $f(x) = \left(\frac{2}{3}\right)^x$

c) $f(x) = .7^x$

d) $f(x) = 4^x$

e) $y = \left(\frac{5}{3}\right)^x$

f) $y = \left(\frac{1}{4}\right)^x$

⑦ Use the above graphs to complete these statements.

* Exponential growth functions have bases that are ______________

* Exponential decay functions have bases that are ______________

⑦ Remember "a", "h" and "k" from when we studied polynomial functions? They work for exponential functions also! ☺

General Form of an Exponential Function:

$$y = a(b)^{x-h} + k$$

b is the base. It determines if the parent is ______ or ______

h determines the shift ____ or ____.

k determines the shift ______ or ______

a determines if the graph ______, ______, or reflects over the __-______.

Complete the following

⑧ $y = 6(3)^{x-1} - 4$

growth or decay? ______

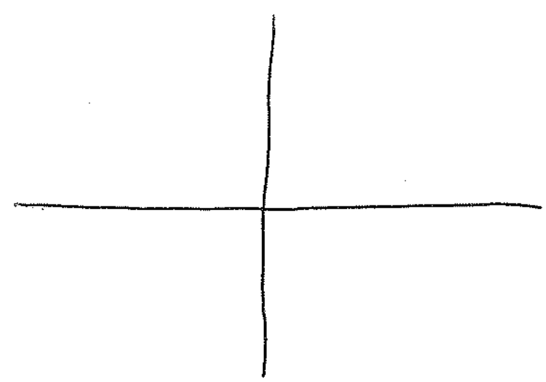
Reflected? ______

Shift L/R? ______

Shift Up/down? ____

y-intercept at (0, ___)

Sketch:



D: ______ R: ______

Incr: ______ Decr: ______

asymptote at ______

⑨ $f(x) = -2\left(\frac{1}{4}\right)^x + 3$

growth or decay? ________

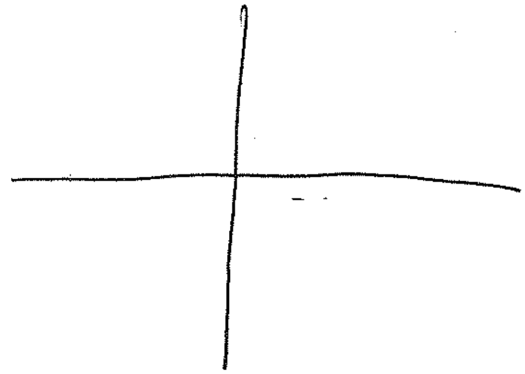
Reflected? ________

Shift L/R? ________

Shift up/down? ________

y-intercept at (0, ___)

Sketch:



D: ________ R: ________

Incr: ________ Decr: ________

asymptote at ________

⑩ $y = -4(2)^{x-2}$

growth or decay? ________

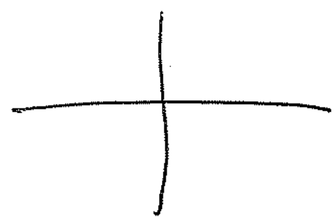
Reflected? ________

Shift L/R? ________

Shift up/down? ________

y-intercept ________

Sketch:



D: ________ R: ________

Incr: ________ Decr: ________

asymptote at ________

⑪ $f(x) = 5\left(\frac{4}{5}\right)^{x+1} + 2$

growth or decay? ________

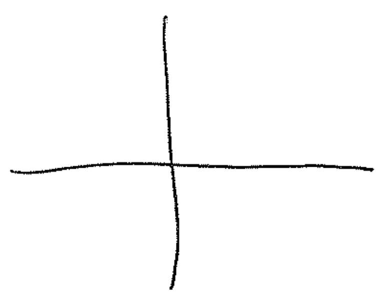
Reflected? ________

Shift L/R? ________

Shift up/down? ________

y-intercept ________

Sketch:



D: ________ R: ________

Incr: ________ Decr: ________

asymptote at ________

10

Day 3 Homework

8.7 Graphing Exponential Functions

The general form of an exponential function is $y = ab^x$ where a is the original amount and b is the growth or decay factor. When $b > 1$, the function models growth and when $0 < b < 1$, the function models decay.

Let's graph functions in the form $y = ab^x$.

1. $y = \frac{1}{2} \cdot 4^x$

x	y
-2	
-1	
0	
1	
2	

2. $y = 2 \cdot 3^x$

x	y
-2	
-1	
0	
1	
2	

3. Compare the graphs from #1 and #2. How are they similar? How do they differ?

4. $y = 3\left(\frac{1}{2}\right)^x$

x	y
-2	
-1	
0	
1	
2	

5. $y = 9\left(\frac{1}{3}\right)^x$

x	y
-2	
-1	
0	
1	
2	

6. Compare the graphs from #4 and #5. How are they similar? How do they differ? Do they represent growth or decay?

7. Which graphs do you think represent growth? Which graphs do you think represent decay?

8. Write the domain and range for each function above.

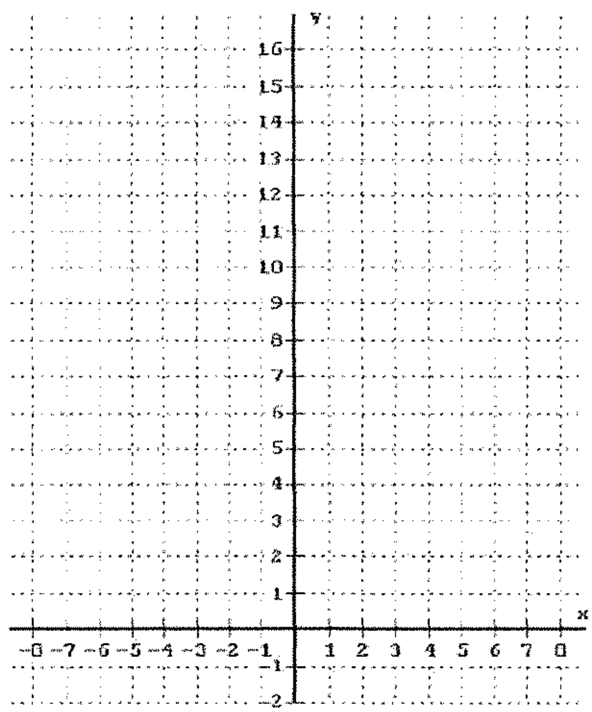
Unit 1 – Day 4 Notes: Translating and Graphing Exponential Functions

Graph the Exponential Function: $f(x) = 2^x$

1. Complete the Table.

x	$y = 2^x$
5	$2^5 = 32$
4	
3	
2	
1	
0	
−1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
−2	
−3	
−4	
−5	
10	
−10	

2. Graph the function.



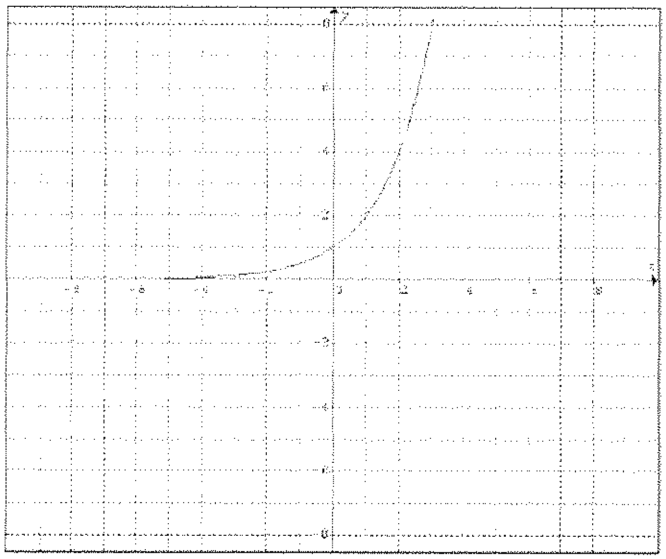
3. Where does the graph of $y = 2^x$ cross the y-axis?
That is, find the y-intercept.

4. Where does the graph of $y = 2^x$ cross the x-axis?
That is, find the x-intercept(s).

5. State the Domain

6. State the Range

Given the graph of the exponential function $f(x) = 2^x$: $f(x) = 2^x$



Graph the following functions on the coordinate plane to the right. After graphing each function, describe the change from the parent function $y = 2^x$.

7. Graph: $y = 2^x - 5$ by making a table. What change occurred from the parent graph $y = 2^x$? Where is the y-intercept now? Where is the x-intercept now?

X-Values	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y-Values											

8. Graph $y = 2^{x-5}$ by making a table. What change occurred from the parent graph $y = 2^x$? Where is the y-intercept now? Where is the x-intercept now?

X-Values	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y-Values											

9. Graph $y = 2^{x-5} - 4$ by making a table. What change occurred from the parent graph $y = 2^x$? Where is the y-intercept now? Where is the x-intercept now?

X-Values	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y-Values											

10. Graph: $y = 2^x + 3$ by making a table. What change occurred from the parent graph $y = 2^x$? Where is the y-intercept now? Where is the x-intercept now?

X-Values	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y-Values											

11. Graph $y = 2^{x+3}$ by making a table. What change occurred from the parent graph $y = 2^x$? Where is the y-intercept now? Where is the x-intercept now?

X-Values	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y-Values											

12. Graph $y = 2^{x+3} + 4$ by making a table. What change occurred from the parent graph $y = 2^x$? Where is the y-intercept now? Where is the x-intercept now?

X-Values	-5	-4	-3	-2	-1	0	1	2	3	4	5
Y-Values											

13. Describe in your own words how to make a horizontal shift with an exponential function. What does the general exponential function look like?

14. Describe in your own words how to make a vertical shift with an exponential function. What does the general exponential function look like?

15. Describe in your own words how to make both a vertical and horizontal shift with an exponential function. What does the general exponential function look like?

Day 4 Homework Sheet 11.4 Name ________

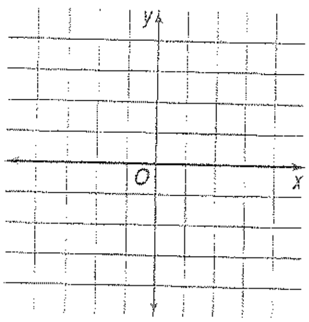
Exponential Functions

Use a calculator to determine the approximate value of each expression to the nearest hundredth.

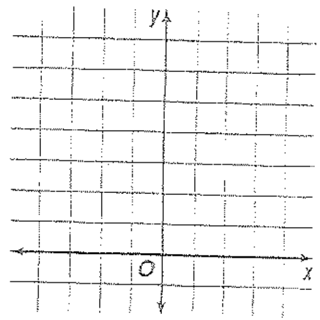
1. $2^{0.7}$ 2. $3^{-0.3}$ 3. $4^{1.1}$ 4. $5^{-4.1}$

Graph each function. State the y-intercept.

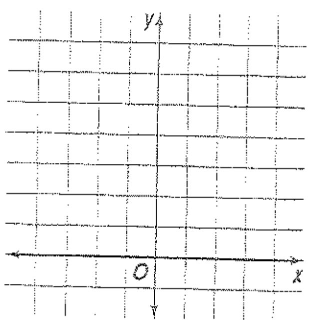
5. $y = 3^x - 2$



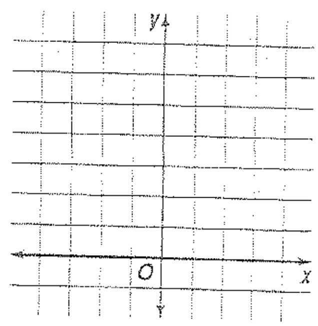
6. $y = 2^{x+1}$



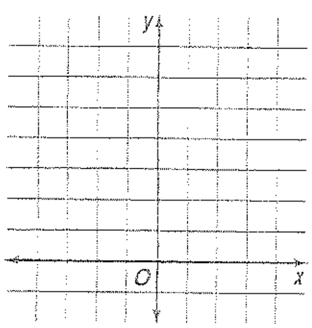
7. $y = 3 \cdot 2^x$



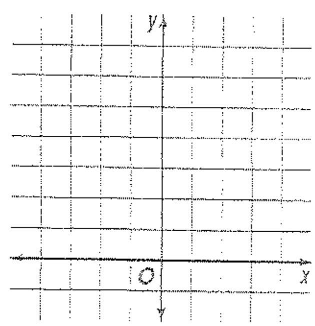
8. $y = \left(\frac{1}{2}\right)^x$



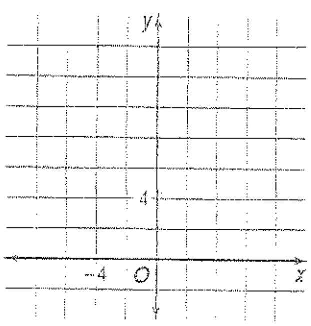
9. $y = 0.3^x$



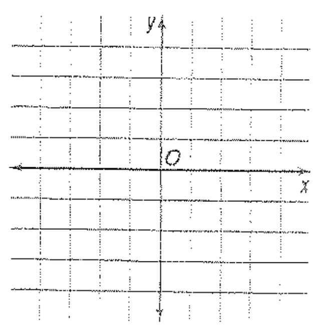
10. $y = 3^x + 1$



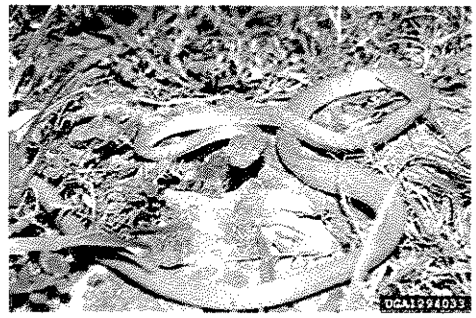
11. $y = 3^{2x-1}$



12. $y = \left(\frac{1}{2}\right)^x - 2$



Unit 1 – Day 5 Notes: Sequences



Vocabulary from this Lesson:
Arithmetic Sequence   Common Difference
Geometric Sequence   Common Ratio
Initial Term

Introduction: The Brown Tree Snake

The Brown Tree Snake is responsible for entirely wiping out over half of Guam's native bird and lizard species as well as two out of three of Guam's native bat species. The Brown Tree Snake was inadvertently introduced to Guam by the US military due to the fact that Guam is a hub for commercial and military shipments in the tropical western Pacific. It will eat frogs, lizards, small mammals, birds and birds' eggs, which is why Guam's bird, lizard, and bat population has been affected. Listed in the table below is the data collected on the Brown Tree Snake's invasion of Guam.

Adapted from Global Invasive Species Database (http://www.issg.org/database/species/ecology.asp?si=54)

The number of snakes for the first few years is summarized by the following sequence:

1, 5, 25, 125, 625, . . .

- What are the next three terms of the sequence?

- How did you predict the number of snakes for the 6^{th}, 7^{th}, and 8^{th} terms?

- What is the initial term of the sequence?

- What is the pattern of change?

Arithmetic Sequences

An _______________ goes from one term to the next by always adding (or subtracting) the same value.

Example: 2, 5, 8, 11, 14, . . . and 7, 3, –1, –5, . . . are arithmetic, since you ______ in the first sequence and __________ in the second sequence, respectively, at each step.

The number added (or subtracted) at each stage of an arithmetic sequence is called the _________________, because if you subtract (find the difference of) successive terms, you'll always get this common value.

Example: find the common difference and the next term of the following sequence:

3, 11, 19, 27, 35, . . .

You Try!!

Ex: Find the common difference and the next term of the following sequence:

1. -4, -7, -10, -13,....

2. 5, 18, 31, 44, 57,....

For arithmetic sequences, the common difference is d, and the first term a_1 is often referred to as the _______________ of the sequence. In the Brown Tree Snake sequence, the rate of change is not arithmetic as shown below.

1, 5, 25, 125, 625, . . .

$5 - 1 = 4$
$25 - 5 = 20$
$125 - 25 = 100$
$625 - 125 = 500$

The difference is not a common number; therefore, the sequence is not arithmetic. So, what kind of sequence is this? Strangely enough, the pattern that I see is one of multiplication.

1, 5, 25, 125, 625, . . .

$1 \bullet 5 = 5$
$5 \bullet 5 = 25$
$25 \bullet 5 = 125$
$125 \bullet 5 = 625$

The initial term of the Brown Tree Snake is _______ and the rate of change is that of _________________ each time in order to generate the next terms of the sequence.

This type of sequence is called a ___________________. A __________________ goes from one term to the next by always multiplying (or dividing) by the same value.

Examples: Are the following sequences arithmetic or geometric?

1. 2, 6, 18, 54,....

2. -7, -9, -11, -13,....

3. 1,2,4,8, 16,.....

The number multiplied (or divided) at each stage of a geometric sequence is called the **common ratio *r***, because if you divide successive terms, you'll always get this common value.
Example: let's determine the common ratio *r* of the Brown Tree Snake Sequence.

1, 5, 25, 125, 625, . . .

The common ratio of the Brown Tree Snake is ____________.

Examples: find the initial term and the common ratio of other geometric sequences.

Example 1: 1/2, 1, 2, 4, 8, . . .

Initial term:________ Common ratio:________

Example 2: 2/9, 2/3, 2, 6, 18, . . .

Initial term:________ Common ratio:________

Classwork: Practice with Sequences

For a sequence, write arithmetic and the common difference or geometric and the common ratio. If a sequence is neither arithmetic nor geometric, write neither.

1) 2, 6, 18, 54, 162, ... ____________________ common __________ = ____

2) 14, 34, 54, 74, 94, ... ____________________ common __________ = ____

3) 4, 16, 36, 64, 100, ... ____________________ common __________ = ____

4) 9, 109, 209, 309, 409, ... ____________________ common __________ = ____

5) 1, 3, 9, 27, 81, ... ____________________ common __________ = ____

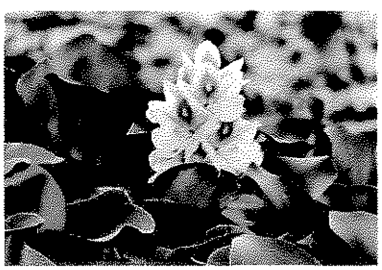
Given the initial term and either common difference or common ratio, write the first 6 terms of the sequence.

6) $a_1 = 7, r = 2$ ____

7) $a_1 = 7, d = 2$ ____

8) $a_1 = 3, r = 5$ ____

9) $a_1 = 4, d = 15$ ____



The water hyacinth is an invasive species from Brazil, which has found its way into North Carolina in the north and inland of the Tar and Neuse river areas. Unchecked, the water hyacinth can lead to clogged waterways, altered water temperature and chemistry, and the exclusion of native plants and wildlife in our own state. Some NC biologists found a region in which 76.9 $miles^2$ were covered by the water hyacinth. They decided to monitor the area by checking it again every 10 days. Here's the data that they collected:

78.9; 157.8; 315.6; 631.2; 1,262.4, . . .

10) Is the area of the plant growing arithmetically or exponentially? Explain how you know by listing the features of the sequence (common difference or common ratio).

11) How is this problem different from the Brown Tree Snake sequence?

Homework: Practice with Geometric Sequences

Determine if the sequence is geometric. If it is, find the common ratio.

1) 56, 28, 14, 7,... 2) 64, -48, 36, -27,... 3) 9, 6, 3, 0, -3, -6, ...

4) 1000, 100, 10, . . . 5) 8, 2, ½, . . . 6) 18, 6, 2, . . .

Given the initial term and common ratio, write the first 6 terms of the sequence.

7) $a_1 = 7, r = 2/3$ ______________________________

8) $a_1 = 5, r = ½$ ______________________________

9) $a_1 = 3, r = 3/5$ ______________________________

10) $a_1 = 3/7, r = ¼$ ______________________________

Problem Situation:

A hot vanilla latte from McDonalds is poured into a cup and allowed to cool while you are riding to school. The difference between the latte temperature and room temperature is recorded every minute for 10 minutes. The sequence is found below:

80, 72, 65, 58, 52, 47, 43, 38, 34, 31, 28

11) Is this sequence geometric? If so, what is the approximate common ratio?

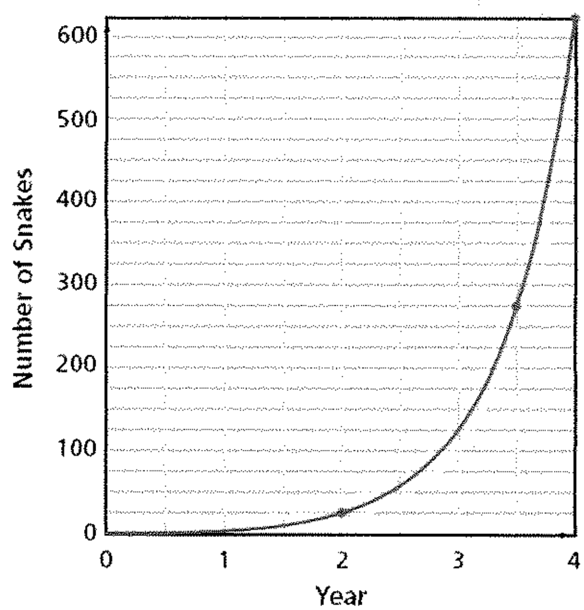
12) How is problem similar or different to the Black Rhinoceros problem in the lesson?

Unit 1 - Day 6 Notes: From Geometric Sequences to Tables and Graphs

The geometric sequence from the Brown Tree Snake problem (1, 5, 25, 125, 625 . . .) can be written in the form of a table, as shown below:

Year	0	1	2	3	4
# of Snakes	1	5	25	125	625

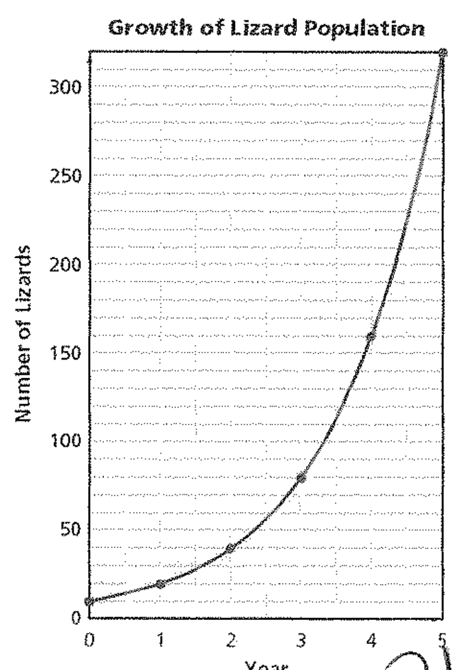
The Brown Tree Snake was first introduced to Guam in year 0. At the end of year 1, five snakes were found; at the end of year 2, twenty-five snakes were discovered, and so on. Since we now have a table of the information, a graph can be drawn, where the year is the independent variable (x) and the number of snakes is the dependent variable (y). See below:



Notice that the graph of the table is not a straight line. Therefore, the graph is not linear in nature, which we already from the fact that the sequence is not arithmetic. Rather, the graph is curved and moves in a growing fashion very rapidly due to the fact that the common ratio *r* of this sequence is 5.

The curved graph of this problem situation is known as an ___________________ function. An ______________ ______________ occurs when the common ratio *r* is ___________________. Tables and graphs make viewing the data from the problem situation easier to see and we can easily see from either the table or graph that in year 3, the snake population is 125.

Let us look at a similar population growth for a certain kind of lizard in both a table and graph. Use either one or both to answer the questions below.



Year	0	1	2	3	4	5
Number of lizards	10	20	40	80	160	320

Notice from the shape of the graph that the information is exponential in nature.

1. What information does the point (2, 40) on the graph tell you?

2. What information does the point (1, 20) on the graph tell you?

3. When will the population exceed 100 lizards?

4. Explain how to find the common ratio, using either the table or graph.

5. If the information from the table were written as a sequence, what is the initial term?

6. How could we find the 10^{th} term in the table, graph, or sequence?

Discuss your answers with your group or with a partner. Now let us explore some more problems.

The Mice Problem

A population of mice has a growth factor (otherwise known as the common ratio) of 3. After 1 month, there are 36 mice. After 2 months, there are 108 mice.

1. How many mice were in the population initially (at 0 months)? Explain how you found this number.

2. Write a sequence to show how the mice population is growing.

3. Is this sequence arithmetic or geometric? Explain how you know.

4. Now, put your sequence into the table below.

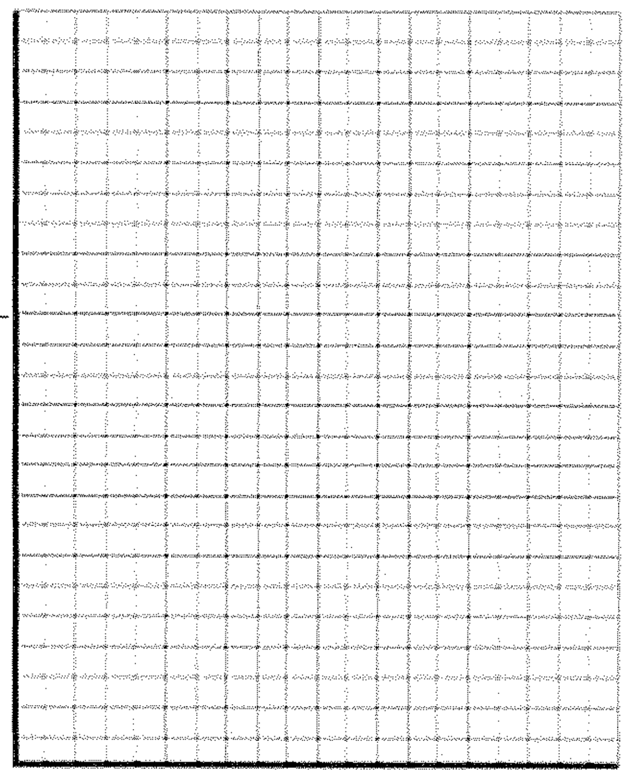
Months	0	1	2	3
Number of Mice				

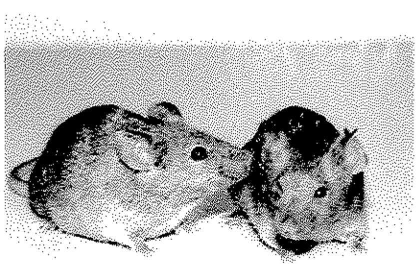
5. Is the graph of the table going to be a straight line or a curve? Explain your answer.

6. Graph the table to make sure of your answer on the graph below. Make sure you label and title the graph below.

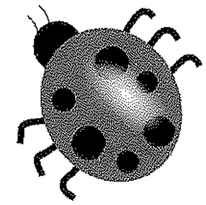
   a. What is your scale for the x-axis?

   b. What is your scale for the y-axis?





Classwork: The Ladybug Invasion



As a biology project, Tamara is studying the growth of a ladybug population. She starts her experiment with 5 ladybugs. The next month she counts 15 ladybugs.

1. Suppose the ladybug population is growing arithmetically. How many beetles can Tamara expect to find after 2, 3, and 4 months? Write the sequence.

2. What is the common difference?

3. Now put the sequence into a table in the space below.

4. How long will it take the ladybug population to reach 200 if it is growing linearly?

5. Suppose the ladybug population is growing exponentially. How many beetles can Tamara expect to find after 2, 3, and 4 months? Write the sequence.

6. What is the common ratio?

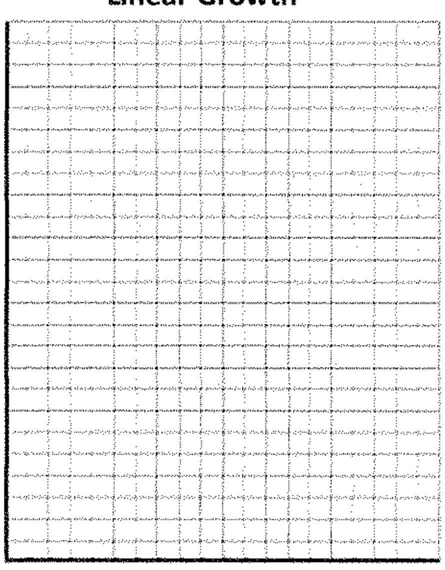
7. Now put the sequence into a table in the space below.

8. How long will it take the ladybug population to reach 200 if it is growing exponentially?

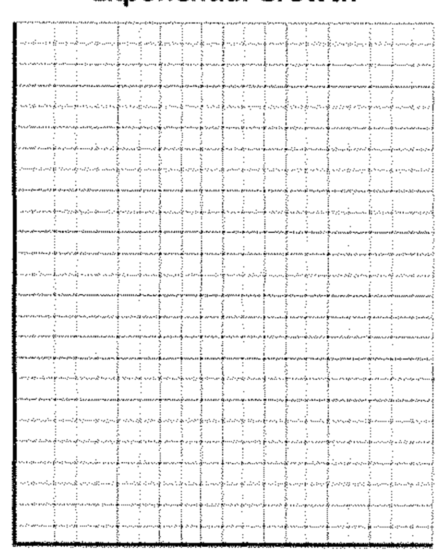
9. Graph both tables on the designated graphs below. Be sure to label your axes.

10. Why does it take the ladybug population longer to reach 200 when it grows linearly?

Linear Growth



Exponential Growth



Charity Donations

Mari's wealthy Great-aunt Sue wants to donate money to Mari's school for new computers. She suggests three possible pans for her donations.

Plan 1: Great-aunt Sue's first plan is give money in the following way: 1, 2, 4, 8, ... She will continue the pattern in this table until day 12. Complete the table to show how much money the school would receive each day.

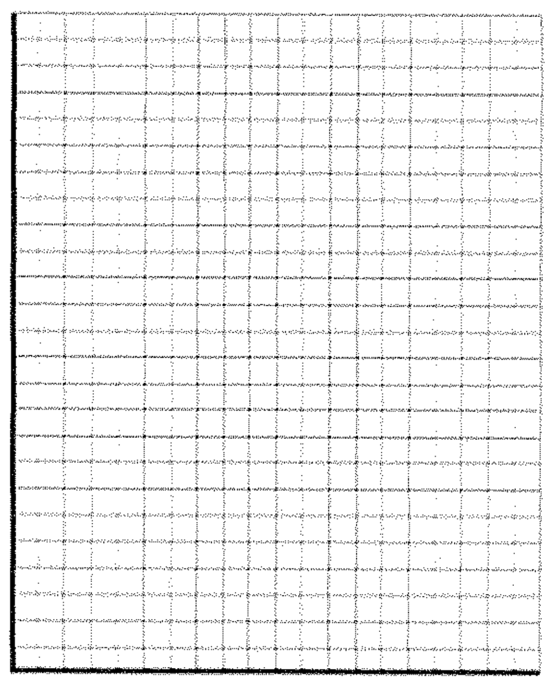
Day	1	2	3	4	5	6	7	8	9	10	11	12
Donation	\$1	\$2	\$4	\$8								

Plan 2: Great-aunt Sue's second plan is to give funds in the following way: 1, 3, 9, 27, ... She will continue the pattern in this table until day 10. Complete the table to show how much money the school would receive each day.

Day	1	2	3	4	5	6	7	8	9	10
Donation	\$1	\$3	\$9	\$27						

Plan 3: Great-aunt Sue's third plan is to give money in the following way: 1, 4, 16, 64, . . . She will continue the pattern in this table until day 7. Complete the table to show how much money the school would receive each day.

Day	1	2	3	4	5	6	7
Donation	\$1	\$4	\$16	\$64			



Graph each plan on the same graph to the right.

1. How much does each plan give the school on day 6?
2. What is the common ratio (growth rate) for each plan?
   a. Plan 1 ___________
   b. Plan 2 ___________
   c. Plan 3 ___________
3. Which plan should the school choose? Why?
4. Which plan will give the school the **greatest total** amount of money?

Homework: Charity Swim-A-Thon

Jason is planning to swim in a charity swim-a-thon. Several relatives have agreed to sponsor him in this charity event. Each of their donations is explained below.

Grandfather: I will give you $1 if you swim 1 lap, $3 if you swim 2 laps, $5 if you swim 3 laps, $7 if you swim 4 laps, and so on.

Father: I will give you $1 if you swim 1 lap, $3 if you swim 2 laps, $9 if you swim 3 lops, $27 if you swim 4 laps, and so on.

Aunt June: I will give you $2 if you swim 1 lap, $3.50 if you swim 2 laps, $5 if you swim 3 laps, $6.50 if you swim 4 laps, and so on.

Uncle Bob: I will give you $1 if you swim 1 lap, $2 if you swim 2 laps, $4 if you swim 3 laps, $8 if you swim 4 laps, and so on.

5. Decide whether each donation sequence is exponential, linear, or neither.

   a. Grandfather's Plan ________________________

   b. Father's Plan ________________________

   c. Aunt June's Plan ________________________

   d. Uncle Bob's Plan ________________________

6. Complete the table for each sequence below.

Grandfather's Plan

# of Laps	1	2	3	4	5	6	7	8	9	10
Donation	$1	$3	$5	$7						

Father's Plan

# of Laps	1	2	3	4	5	6	7	8	9	10
Donation	$1	$3	$9	$27						

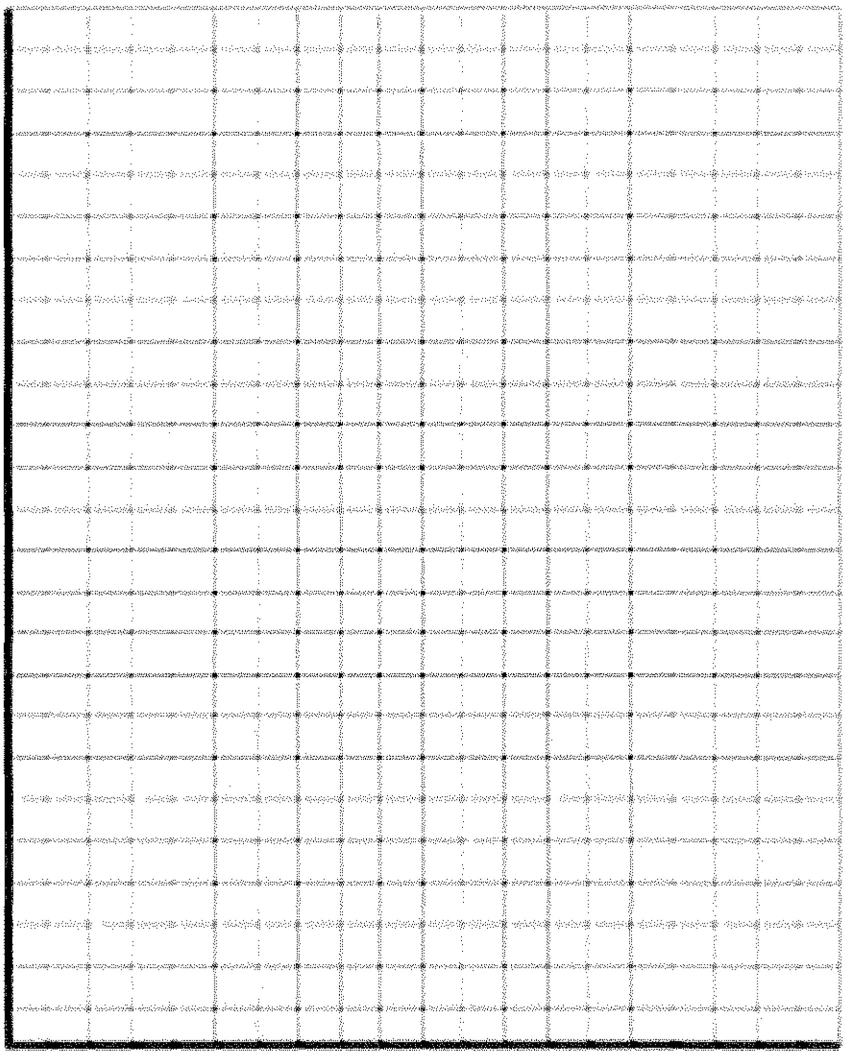
Aunt June's Plan

# of Laps	1	2	3	4	5	6	7	8	9	10
Donation	$2	$3.50	$5	$6.50						

Uncle Bob's Plan

# of Laps	1	2	3	4	5	6	7	8	9	10
Donation	$1	$2	$4	$8						

7. Graph each table on the graph below. Label each line or curve. Title the graph and label the axes.



8. Use either the table or graph to determine the **total money** Jason will raise for each plan if he swims 10 laps.
   a. Grandfather's Plan ______________________________
   b. Father's Plan ______________________________
   c. Aunt June's Plan ______________________________
   d. Uncle Bob's Plan ______________________________

Adapted from *Growing, Growing, Growing, Exponential Relationships*, Connected Mathematics 2, Pearson, 2009.

Classwork: One Grain of Rice

NAME ____________________

Common Ratio	NOW-NEXT

In the book *One Grain of Rice* by Demi, the main character Rani cleverly tricks the raja into giving rice to the village. Use the story from the book to answer the questions below.

1. Estimate how many grains of rice you think Rani will have at the end of 30 days.

2. Use the chart below to record the number of grains of rice Rani would receive each day.

Day 1 **1** *grain of rice*	*Day 2* **2** *grain of rice*	*Day 3* *grain of rice*	*Day 4* *grain of rice*	*Day 5* *grain of rice*	***Total After 5 Days***
Day 6 *grain of rice*	*Day 7* *grain of rice*	*Day 8* *grain of rice*	*Day 9* *grain of rice*	*Day 10* **512** *grain of rice*	***Total After 10 Days***
Day 11 *grain of rice*	*Day 12* *grain of rice*	*Day 13* *grain of rice*	*Day 14* *grain of rice*	*Day 15* *grain of rice*	***Total After 15 Days***
Day 16 *grain of rice*	*Day 17* *grain of rice*	*Day 18* **131,072** *grain of rice*	*Day 19* *grain of rice*	*Day 20* *grain of rice*	***Total After 20 Days***
Day 21 *grain of rice*	*Day 22* *grain of rice*	*Day 23* *grain of rice*	*Day 24* *grain of rice*	*Day 25* *grain of rice*	***Total After 25 Days***
Day 26 *grain of rice*	*Day 27* *grain of rice*	*Day 28* *grain of rice*	*Day 29* *grain of rice*	*Day 30* *grain of rice*	***Total After 30 Days***

3. If the story continued and you know how many grains of rice Rani receives on Day 30, how can you determine how many grains of rice she would receive on Day 31?

4. How can you determine how many grains of rice she would receive on Day 35?

5. How can you determine how many grains of rice she would receive on Day 40?

6. If you know how many grains of rice she receives on a certain day, how can you determine how many grains of rice she will receive 2 days later? . . . 10 days later?

7. Write a recursive rule that describes how many grains of rice Rani receives each day.

Homework: The Million Dollar Mission

Name ______________________________

You're sitting in math class, minding your own business, when in walks a Bill Gates kind of guy - the real success story of your school. He's made it big, and now he has a job offer for you.
He doesn't give too many details, mumbles something about the possibility of danger. He's going to need you for 30 days, and you'll have to miss school. (Won't that just be too awful?) And you've got to make sure your passport is current. (Get real, Bill, this isn't Paris). But do you ever sit up at the next thing he says:

You'll have your choice of two payment options:

1. One cent on the first day, two cents on the second day, and double your salary every day thereafter for the thirty days; or
2. Exactly $1,000,000. (That's one million dollars!)

You jump up out of your seat at that. You've got your man, Bill, right here. You'll take that million. You are there. And off you go on this dangerous million-dollar mission.

So how smart was this guy? Did you make the best choice? Before we decide for sure, let's investigate the first payment option. Complete the table for the first week's work.

First Week – First Option

Day No.	Pay for that Day	Total Pay (In Dollars)
1	.01	.01
2	.02	.03
3		
4		
5		
6		
7		

So, after a whole week you would have only made _____________.

That's pretty awful, all right. There's no way to make a million in a month at this rate. Right? Let's check out the second week. Complete the second table.

Second Week – First Option

Day No.	Pay for that Day	Total Pay (In Dollars)
8		
9		
10		
11		
12		
13		
14		

Well, you would make a little more the second week; at least you would have made _____________ _____. But there's still a big difference between this salary and $1,000,000. What about the third week?

Third Week – First Option

Day No.	Pay for that Day	Total Pay (In Dollars)
15		
16		
17		
18		
19		
20		
21		

We're getting into some serious money here now, but still nowhere even close to a million. And there's only 10 days left. So it looks like the million dollars is the best deal. Of course, we suspected that all along.

Fourth Week – First Option

Day No.	Pay for that Day	Total Pay (In Dollars)
22		
23		
24		
25		
26		
27		
28		

Hold it! Look what has happened. What's going on here? This can't be right. This is amazing. Look how fast this pay is growing. Let's keep going. I can't wait to see what the total will be.

Last 2 Days – First Option

Day No.	Pay for that Day	Total Pay (In Dollars)
29		
30		

In 30 days, it increases from 1 penny to over ______________________________dollars. That is absolutely amazing.

Questions to consider:

1. How can you determine how much money he would receive on Day 35?

2. How can you determine how much money he would receive on Day 40?

3. If you know how much money he receives on a certain day, how can you determine how much money he will receive 2 days later? . . . 10 days later?

4. Write a sentence that describes how much money the guy receives each day.

5. What is the rate of change? Is this a common difference or a common ratio?

6. Use the words *NOW* and *NEXT* to write a rule to express the pattern.

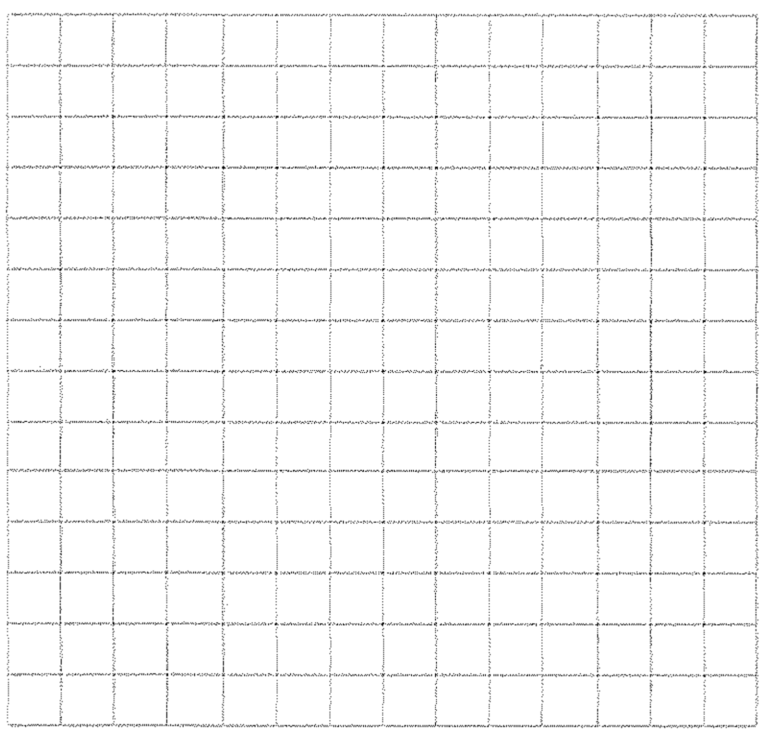
7. How do the numbers and calculations used in your NOW-NEXT rule express the pattern of change in the first option salary table (number of day, amount of money)?

8. Write a sentence that describes the total amount of money the guy will receive through a certain number of days.

9. Use the words NOW-NEXT to write a rule to describe the total amount of pay for a particular day.

10. Test each of your equations to see if they generate the values in the table. Were your algebraic equations correct? If not, modify your equations and test them until you are certain they are correct. Record the changes you make so that you can explain to others how you arrived at your final equations.

11. Graph the first ten days of salary option 1 on the graph to the right. Be sure to label your axes and title your graph. Is this a graph of a linear function or an exponential function?



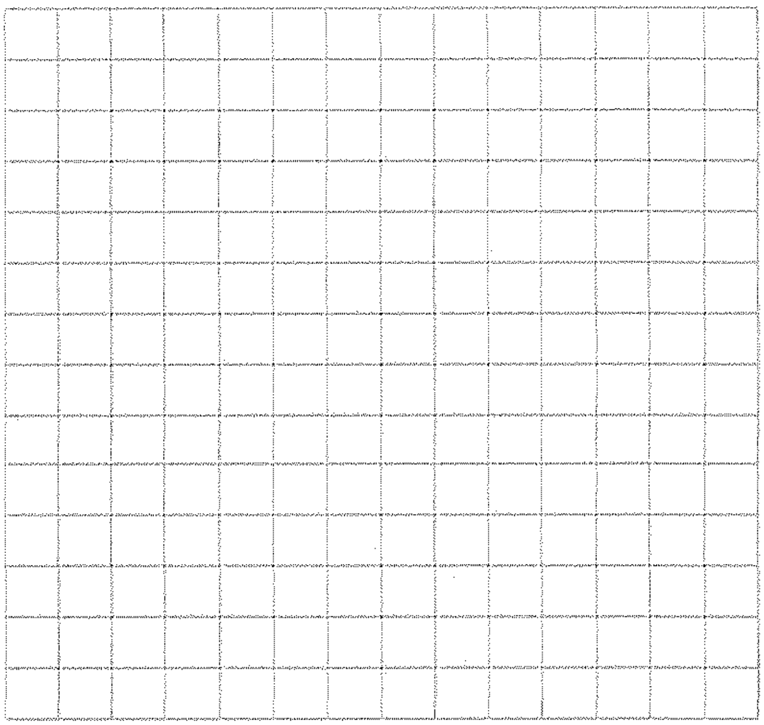
Unit 1 – Day 9 Notes: Bacteria Growth

If you don't brush your teeth regularly, it won't take long for large colonies of bacteria to grow in your mouth. Suppose a single bacterium lands on your tooth and starts multiplying by a factor of 4 every hour.

1. Complete the table below to model the bacteria growth over several hours.

Hours	0	1	2	3	4	5
Number of Bacteria	1	4	16			

2. Graph the data in the table below. Be sure to label your graph and axes.



3. Is this graph linear or exponential?

4. Write the *NOW-NEXT* form to show the pattern of growth.

   NEXT = _______ • *NOW*

5. What is the common ratio *r*?

6. Use the common ratio *r* to write a rule to showing how to calculate the number of bacteria *y* after *x* hours.

   y = the number of bacteria produced in that hour
   x = the number of hours
   r = the common ratio or rate of change
   a_1 = the initial term of the sequence or the starting point

   Use the above information to write the explicit form of the exponential function

$y = a_1 \cdot r^x$. Notice how similar it is to the NOW-NEXT recursive form.

$$NEXT = NOW \cdot r$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$y = a_1 \cdot r^x$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$y = 1 \cdot 4^x$$

The *NEXT* and *y* components both represent the number of bacteria generated during the hour. The *NOW* and a_1 both represent the starting point and *r* is the rate of change or the common ratio, which is 4 in this example.

7. Use the rule in step 6 to determine the number of bacteria in the colony after 7 hours. Verify the number of bacteria by either continuing the table in step 1 or continuing the graph in step 2.

8. After how many hours will there be at least 1,000,000 bacteria in the colony?

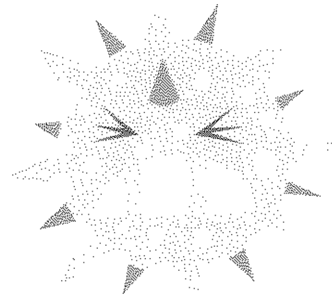
9. Suppose that instead of 1 bacterium, 50 bacteria land in your mouth. Write an explicit equation which describes the number of bacteria *y* in this colony after *x* hours.

10. What is different in this equation from the equation in step 6?

11. Using your new equation, determine the number of bacteria in the colony after 8 hours and after 10 hours.

12. Which method for determining the number of bacteria is easier for you? Using a table, graph, NOW-NEXT, or equation? Explain.

Classwork: More Bacteria

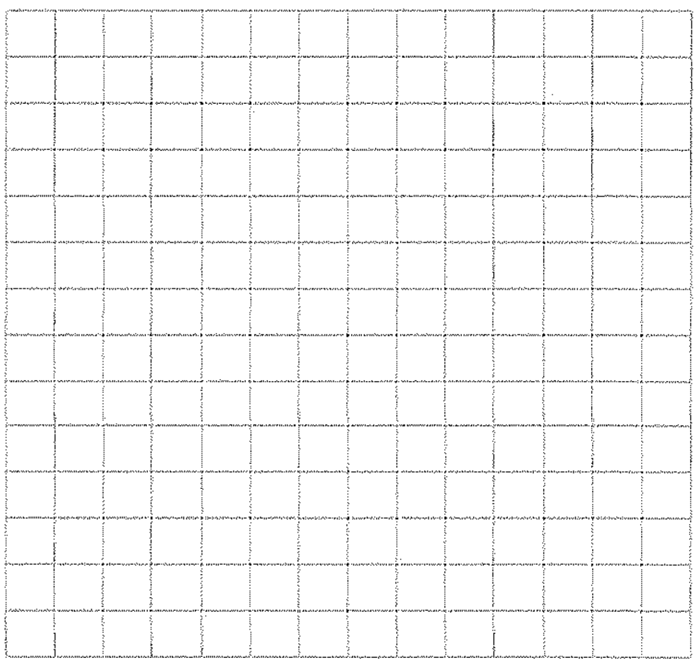


The bacteria E. coli often causes illness among people who eat the infected food. Suppose a single E. coli bacterium in a batch of ground beef begins doubling every 10 minutes.

1. Complete the table below to determine how many bacteria there will be after 10, 20, 30, 40, and 50 minutes have elapsed (assuming no bacteria die).

10-min Period	1	2	3	4	5
Number of Bacteria	2				

2. Graph the data on the table. Be sure to title your graph and label your axes.



3. Write two rules that can be used to calculate the number of bacteria in the food after any number of 10-minute periods.

NEXT = NOW • ___

↓ ↓ ↓

$y = a_1 \cdot r^x$

↓ ↓ ↓

y = ___ • ___ ^{x}

4. What is the initial value?

5. What is the common ratio?

6. Use your rule(s) to determine the number of bacteria after 2 hours.

7. When will the number of bacteria reach 100,000?

Students at a high school conducted an experiment to examine the growth of mold. They set out a shallow pan containing a mixture of chicken broth, gelatin, and water. Each day, the students recorded the area of the mold in square millimeters. The students wrote the exponential equation $m = 50(3^d)$ to model the growth of the mold. In this equation, m is the area of the mold in square millimeters after d days.

8. What is the area of the mold at the start of the experiment?

9. What is the growth factor or common ratio?

10. What is the area of the mold after 5 days?

11. On which day will the area of the mold reach 6,400 mm^2?

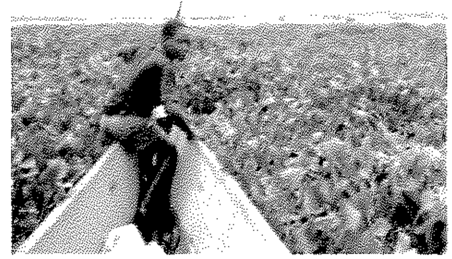
12. An exponential equation can be written in the form $y = a(b^x)$, where a and b are constant values.

    a. What value does b have in the mold equation? What does this value represent?

    b. What value does a have in the mold equation? What does this value represent?

Lesson adapted from *Growing, Growing, Growing Exponential Relationships*, Connected Mathematics 2, Pearson, 2009.

Homework: Killer Plants

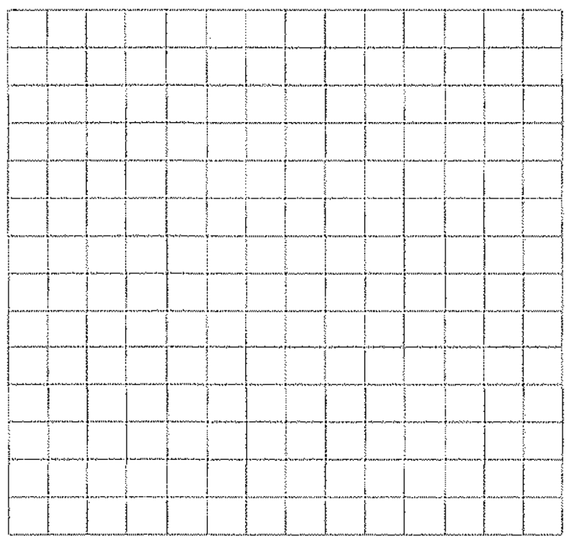


Ghost Lake is a popular site for fishermen, campers, and boaters. In recent years, a certain water plant has been growing on the late at an alarming rate. The surface area of Ghost Lake is 25,000,000 square feet. At present, 1,000 square feet are covered by the plant. The Department of Natural Resources estimates that the area is doubling every month.

1. Complete the table below.

Number of Months	1	2	3	4	5
Area Covered in Square Feet	1,000				

2. Use the data to graph the situation. Be sure to label your axes and title your graph.



3. Write 2 equations (NOW-NEXT and y =) to represent the growth pattern of the plant on Ghost Lake.

4. Explain what information the variables and numbers in your equations represent.

5. How much of the lake's surface will be covered with the water plant by the end of a year?

6. In how many months will the plant completely cover the surface of the lake?

Loon Lake has a "killer plant" problem similar to Ghost Lake. Currently, 5,000 square feet of the lake is covered with the plant. The area covered is growing by a factor of 1.5 each year.

7. Complete the table to show the area covered by the plant for the next 5 years.

Number of Years	1	2	3	4	5
Area Covered in Square Feet	5,000				

8. Graph the data. Be sure to label your axes and title your graph.

9. Write 2 equations (NOW-NEXT and y =) to represent the growth pattern of the plant on Ghost Lake.

10. Explain what information the variables and numbers in your equations represent.

11. How much of the lake's surface will be covered with the plant by the end of 7 years?

12. The surface area of the lake is approximately 200,000 square feet. How long will it take before the lake is completely covered?

Adapted from Growing, Growing, Growing Exponential Relationships, Connected Mathematics 2, Pearson, 2009.